International Journal of Emerging Engineering Research and Technology
Volume 4, Issue 11, November 2016, PP 37-47
ISSN 2349-4395 (Print) & ISSN 2349-4409 (Online)

Excitation Controlers Disign of a Hydrogenerator Power Unit based on Multirate Sampling

A. K. Boglou[1], D. I. Pappas[1], S. K. Goumas[2]

[1]Department of Petroleum and Mechanical Engineering, Eastern Macedonia and Thrace Institute of Technology, Kavala, Greece
[2]School of Business and Economics, Eastern Macedonia and Thrace Institute of Technology, Kavala, Greece

ABSTRACT

In this paper an H^{∞} -control technique is presented and applied to the design of optimal multirate-output controllers. The technique is based on multirate-output controllers (MROCs) having a multirate sampling mechanism with different sampling period in each measured output of the system. It relies mainly on the reduction , under appropriate conditions, of the original H^{∞} -disturbance attenuation problem to an associated discrete H^{∞} -control problem for which a fictitious static state feedback controller is to be designed, even though some state variables are not available (measurable) for feedback purposes. The proposed H^{∞}-control technique is applied to the discrete linear open-loop system model which represents a 117 MVA synchronous generator with conventional exciter supplying line to an infinite grid show the effectiveness of the proposed method which has a quite satisfactory performance

Keywords: H^{∞} -control, disturbance, multirate control, power system.

INTRODUCTION

The H^{∞} -optimization control problem has drawn great attention [1-6]. In particular, the H^{∞} -control problem for discrete-time and sampled-data singlerate and multirate systems has been treated successfully [7-9, 11-15]. Generally speaking, when the state vector is not available for feedback, the H^{∞} -control problem is usually solved in both the continuous and discrete-time cases using dynamic measurement feedback approach.

Recently, a new technique [9, 10] is presented for the solution of the H^{∞} -disturbance attenuation problem. This technique is based on multirate-output controllers (MROCs) and in order to solve the sampled-data H^{∞} -disturbance attenuation problem relies mainly on the reduction, under appropriate conditions, of the original H^{∞} -disturbance attenuation problem, to an associated discrete H^{∞} -control problem for which a fictitious static state feedback controller is to be designed, even though some state variables are not available for feedback.

In the present work, the design of the governor for the disturbance attenuation of an hydrogenerator power unit is developed, by using the same methodology as in the case of [6], but improved in the aspect of using much less state feedback measurements, for the same case study power unit. Feedback measurements consist of (a) the torque angle, (b) the machine speed and (c) the exciter output voltage. The linearized, continuous, 6th -order SIMO open-loop model representing a practical power system with impulse disturbances (composed of a 117 MVA hydrogenerator supplying power to an infinite grid through a proper connection network [16-19]) is used. The digital controller, which leads to the associated designed discrete closed-loop power system model, achieved enhanced dynamic stability characteristics. This is accomplished by applying the presented multirate-output controller technique, based on H^{∞} optimization control.

OVERVIEW OF RELEVANT MATHEMATICAL CONSIDERATIONS

The general description of the controllable and observable continuous, linear, time-invariant, multivariable MIMO dynamical open-loop system expressed in state-space form is

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) \tag{1}$$

where: $\mathbf{x}(t) \in \mathbf{R}^n$, $\mathbf{u}(t) \in \mathbf{R}^m$, $\mathbf{y}(t) \in \mathbf{R}^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

The associated general discrete description of the system (1) is as follows

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$
$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \tag{2}$$

where: $\mathbf{x}(k) \in \mathbf{R}^n$, $\mathbf{u}(k) \in \mathbf{R}^m$, $\mathbf{y}(k) \in \mathbf{R}^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

OVERVIEW OF H$^\infty$-CONTROL TECHNIQUE USING MULTIRATE-OUTPUT CONTROLLERS (MORCS)

Consider the controllable and observable continuous linear state-space system model of the general form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{D}\mathbf{q}(t), \quad \mathbf{x}(0) = \mathbf{0} \tag{3a}$$

$$\mathbf{y}_m(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{J}_1\mathbf{u}(t),$$

$$\mathbf{y}_c(t) = \mathbf{E}\mathbf{x}(t) + \mathbf{J}_2\mathbf{u}(t) \tag{3b}$$

where: $\mathbf{x}(t) \in \mathbf{R}^n$, $\mathbf{u}(t) \in \mathbf{R}^m$, $\mathbf{q}(t) \in \mathbf{L}_2^d$, $\mathbf{y}_m(t) \in \mathbf{R}^{p_1}$, $\mathbf{y}_c(t) \in \mathbf{R}^{p_2}$ are the state, input, external disturbance, measured output and controlled output vectors, respectively. In (3), all matrices have real elements and appropriate dimensions. Now follows a useful definition.

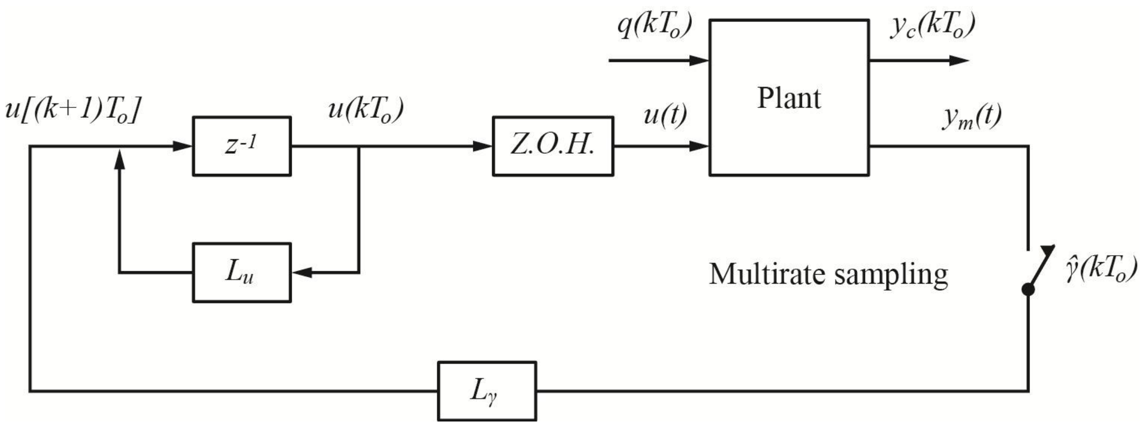


Figure1. *Control of linear systems using MROCs.*

Definition. For an observable matrix pair $(\mathbf{A}, \mathbf{C})$, with $\mathbf{C}^T = [\mathbf{c}_1^T \ \ \mathbf{c}_2^T \ \ \cdots \ \ \mathbf{c}_{p_1}^T]$ and c_i with i=1, …,p1, the ith row of the matrix C, a collection of p_1 integers $\{n_1, n_2, \cdots, n_{p_1}\}$ is called an observability index vector of the pair $(\mathbf{A}, \mathbf{C})$, if the following relationships simultaneously hold

$$\sum_{i=1}^{p_1} n_i = n, \quad \operatorname{rank}\left[\mathbf{c}_1^T \ \cdots \ (\mathbf{A}^T)^{n_1-1}\mathbf{c}_1^T \ \cdots \ \mathbf{c}_{p_1}^T \ \cdots \ (\mathbf{A}^T)^{n_{p_1}-1}\mathbf{c}_{p_1}^T\right] = n$$

Next, the multirate sampling mechanism [14] is applied to system (3).

Assuming that all samplers start simultaneously at t = 0, a sampler and a zero-order hold with period T_0 is connected to each plant input $u_i(t)$, i=1,2,…,m, such that

$$\mathbf{u}(t)=\mathbf{u}^{(kT_0)},\ t \in [kT_0,\ (k+1)T_0) \tag{4}$$

while the ith disturbance $q_i(t)$, i=1,…,d, and the ith controlled output $y_{c,i}(t)$, i=1,…, p_2, are detected at time kT_0, such that for $t \in [kT_0,\ (k+1)T_0)$

$$\mathbf{q}(t)=\mathbf{q}^{(kT_0)},\ \mathbf{y}_c(kT_0)=\mathbf{E}\mathbf{x}^{(kT_0)}+\mathbf{J}_2{}^{(kT_0)} \tag{5}$$

The ith measured output $y_{m,i}(t)$, i=1,…, p_1, is detected at every T_i period, such that for $\mu = 0,\dots, N_i - 1$

$$y_{m,i}(kT_0+\mu T_i)=\mathbf{c}_i\mathbf{x}(kT_0+\mu T_i)+(\mathbf{J}_1)_i\,\mathbf{u}(kT_0) \tag{6}$$

where $(\mathbf{J}_2)_i$ is the ith row of the matrix $\mathbf{J}_2$. Here $N_i \in Z^+$ are the output multiplicities of the sampling and $T_i \in R^+$ are the output sampling periods having rational ratio, i.e. $T_i = T_0 / N_i$ with i=1,…, p_1.

The sampled values of the plant measured outputs obtained over $[kT_0,(k+1)T_0)$ are stored in the N^*-dimensional column vector given by

$$\hat{\gamma}(kT_0)=\big[\,y_{m,1}(kT_0)\ \cdots\ y_{m,1}(kT_0+(N_1-1)T_1)$$
$$\cdots\ \ y_{m,p_1}(kT_0)\ \cdots\ y_{m,p_1}\left[kT_0+\left(N_{p_1}-1\right)T_{p_1}\right]\big]^T \tag{7}$$

(where $N^* = \sum_{i=1}^{p_1} N_i$), that is used in the MOC of the form

$$\mathbf{u}\left[(k+1)T_0\right]=\mathbf{L}_{\mathbf{u}}\mathbf{u}(kT_0)-\mathbf{L}_{\gamma}\hat{\gamma}(kT_0) \tag{8}$$

where $L_{\mathbf{u}} \in R^{m\times m}$, $L_{\gamma} \in R^{m\times N^*}$.

The H^∞-disturbance attenuation problem treated in this paper, is as follows: Find a MROC of the form (4), which when applied to system (3), asymptotically stabilizes the closed-loop system and simultaneously achieves the following design requirement

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty \le \gamma \tag{9}$$

for a given $\gamma \in \mathbf{R}^+$, where $\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty$ is the H∞-norm of the proper stable discrete transfer function $\mathbf{T}_{\mathbf{qy}_c}(z)$, from sampled-data external disturbances $\mathbf{q}(kT_0) \in \ell_2^d$ to sampled-data controlled outputs $\mathbf{T}_{\mathbf{qy}_c}(z)$, defined by

$$\left\|\mathbf{T}_{\mathbf{qy}_c}(z)\right\|_\infty=\sup_{\mathbf{q}(kT_0)\in l_2}\frac{\left\|\mathbf{y}_c(kT_0)\right\|_2}{\left\|\mathbf{q}(kT_0)\right\|_2}=\sup_{\theta\in[0,2\pi]}\sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(e^{j\theta})\right]=\sup_{|z|=1}\sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right] \tag{10}$$

where, $\sigma_{\max}\left[\mathbf{T}_{\mathbf{qy}_c}(z)\right]$ is the maximum singular value of $\mathbf{T}_{\mathbf{qy}_c}(z)$, and where was used the standard definition of the ℓ_2-norm of a discrete signal $\mathbf{s}(kT_0)$

$$\left\|\mathbf{s}(kT_0)\right\|_2^2=\sum_{k=0}^{\infty}\mathbf{s}^T(kT_0)\mathbf{s}(kT_0) \tag{11}$$

Our attention will now be focused on the solution of the above H^∞-control problem. To this end, the following assumptions on system (3) are made:

Assumptions

a) The matrix triplets $(\mathbf{A},\mathbf{B},\mathbf{C})$ and $(\mathbf{A},\mathbf{D},\mathbf{E})$ are stabilizable and detectable.

b) $\operatorname{rank}\begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1 x d} \end{bmatrix} = n + d, \quad \operatorname{rank}\begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{D} \\ \mathbf{C} & \mathbf{0}_{p_1 x m} & \mathbf{0}_{p_1 x d} \end{bmatrix} = n + m + d$

c) $\mathbf{J}_2^T [\mathbf{E} \quad \mathbf{J}_2] = [\mathbf{0}_{m \times n} \quad \mathbf{I}_{m \times m}]$

d) There is a sampling period T_0, such that the open-loop discrete-time system model in general form becomes

$$\begin{aligned} &\mathbf{x}\left[(k+1)T_0\right] = \Phi\,\mathbf{x}(kT_0) + \hat{\mathbf{B}}\,\mathbf{u}(kT_0) + \hat{\mathbf{D}}\,\mathbf{q}(kT_0) \\ &\mathbf{y}_c(kT_0) = \mathbf{E}\,\mathbf{x}(kT_0) + \mathbf{J}_2\mathbf{u}(kT_0) \end{aligned} \tag{12}$$

where $\mathbf{\Phi} = \exp(\mathbf{A}T_0)$, $(\hat{\mathbf{B}}, \hat{\mathbf{D}}) = \int_0^{T_0} \exp(\mathbf{A}\lambda)(\mathbf{B},\mathbf{D})\,d\lambda$ is stabilizable and observable and does not have invariant zeros on the unit circle.

From the above it follows that the procedure for H^∞ -disturbance attenuation using MROCs essentially consists in finding for the control law a fictitious state matrix **F**, which equivalently solves the problem and then, either determining the MROC pair $(\mathbf{L}_\gamma, \mathbf{L}_u)$ or choosing a desired $\mathbf{L}_u$ and determining the $\mathbf{L}_\gamma$. As it has been shown in [2], matrix **F** takes the form

$$\mathbf{F} = \left(\mathbf{I} + \hat{\mathbf{B}}^T \mathbf{P} \hat{\mathbf{B}}\right)^{-1} \hat{\mathbf{B}}^T \mathbf{P} \mathbf{\Phi} \tag{13}$$

where **P** is an appropriate solution of the following Riccati equation

$$\begin{aligned} &\mathbf{P} = \mathbf{E}^T\mathbf{E} + \mathbf{\Phi}^T\mathbf{P}\mathbf{\Phi} - \mathbf{\Phi}^T\mathbf{P}\hat{\mathbf{B}}\left(\mathbf{I} + \hat{\mathbf{B}}^T\mathbf{P}\hat{\mathbf{B}}\right)^{-1}\hat{\mathbf{B}}\mathbf{P}\mathbf{\Phi} + \mathbf{P}\hat{\mathbf{D}}_\gamma\left(\mathbf{I} + \hat{\mathbf{D}}_\gamma^T\mathbf{P}\hat{\mathbf{D}}_\gamma\right)\hat{\mathbf{D}}_\gamma^T\mathbf{P}, \\ &\hat{\mathbf{D}}_\gamma = \gamma^{-1}\hat{\mathbf{D}} \end{aligned} \tag{14}$$

Once matrix **F** is obtained the MROC matrices $\mathbf{L}_\gamma$ and $\mathbf{L}_u$ (in the case where $\mathbf{L}_u$ is free), can be computed according to the following mathematical expressions

$$\begin{aligned} &\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{0}_{m \times d}]\tilde{\mathbf{H}} + \Lambda\left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \Theta_q]\tilde{\mathbf{H}}\right) \\ &\mathbf{L}_u = \left\{[\mathbf{F} \quad \mathbf{0}_{m \times d}]\tilde{\mathbf{H}} + \Lambda\left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \Theta_q]\tilde{\mathbf{H}}\right)\right\}\Theta_u \end{aligned} \tag{15}$$

where $\tilde{\mathbf{H}}[\mathbf{H} \quad \mathbf{\Theta}_q] = \mathbf{I}$ and $\Lambda \in R^{mxN^*}$ is an arbitrary specified matrix. In the case where $\mathbf{L}_u = \mathbf{L}_{u,sp}$, we have

$$\mathbf{L}_\gamma = [\mathbf{F} \quad \mathbf{L}_{u,sp} \quad \mathbf{0}_{m \times d}]\hat{\mathbf{H}} + \mathbf{\Sigma}\left(\mathbf{I}_{N^* \times N^*} - [\mathbf{H} \quad \mathbf{\Theta}_u \quad \mathbf{\Theta}_q]\hat{\mathbf{H}}\right)$$

where $\hat{\mathbf{H}}[\mathbf{H} \quad \mathbf{\Theta}_u \quad \mathbf{\Theta}_q] = \mathbf{I}$ and $\Sigma \in R^{mxN^*}$ is arbitrary.

The resulting closed-loop system matrix $(A_{cl/d})$ takes the following general form

$$\mathbf{A}_{cl/d} = \mathbf{A}_{ol/d} - \mathbf{B}_{ol/d}\mathbf{F} \tag{16}$$

where **cl** = closed-loop, **ol** = open-loop and **d** = discrete.

MODELING AND SIMULATIONS OF HYDROGENERATOR

The system under study, Figure 2, is composed of a 117 MVA hydrogenarator connected to an infinite grid through a step-up transformer and a double-circuit transmission line. The data of the system of Figure 2 [16, 17] are given in Appendix B.

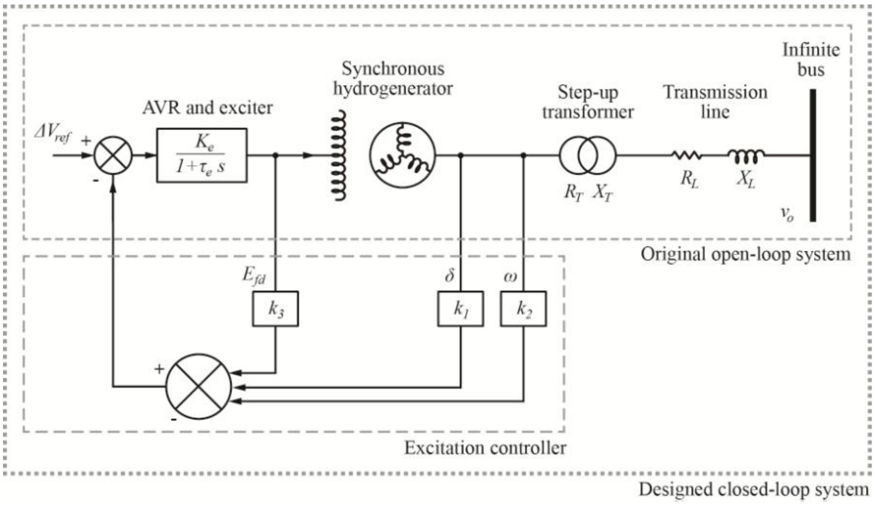


Figure2. *Simplified representation of hydrogenerator system supplying power to an infinite grid.*

In such power system it is customary to start from the standard Park's non-linear equations [18] and after a process of linearization with respect to a nominal operating point (Appendix A, Table 3) in this case [16-17] to represent the system in state space form as in (1):

$$\mathbf{x} = \begin{bmatrix} \Delta\delta & \Delta\omega & \Delta\Psi_f & \Delta\Psi_D & \Delta\Psi_Q & \Delta E_{fd} \end{bmatrix}^T$$

$$\mathbf{u} = \Delta \mathbf{V}_{\mathbf{ref}}, \quad \mathbf{q} = \mathbf{u}, \quad \mathbf{y}_{\mathbf{m}} = \begin{bmatrix} \Delta\delta & \Delta\omega & \Delta E_{fd} \end{bmatrix}^T, \quad \mathbf{y}_{\mathbf{c}} = \mathbf{x},$$

$$\mathbf{E} = \mathbf{I}_{6x6}, \quad \mathbf{J}_1 = \mathbf{0}_{6x1}, \quad \mathbf{J}_2 = \mathbf{0}_{6x1}.$$

The matrices **A, B, C** and **D** are given in Appendix B. Note also that, the states ψ_f, ψ_D and ψ_Q are not measurable quantities.

The eigenvalues of the original continuous open-loop power system models and the simulated responses of the output variables $(\Delta\delta, \Delta\omega, \Delta E_{fd})$, are shown in Table 1 and Figure 3, respectively.

Table1. *Eigen values of original open-loop power system models.*

Original open-loop power system model	λ	-25.7163 -1.5760±j9.0773 -7.7690±j5.9416 -3.6616

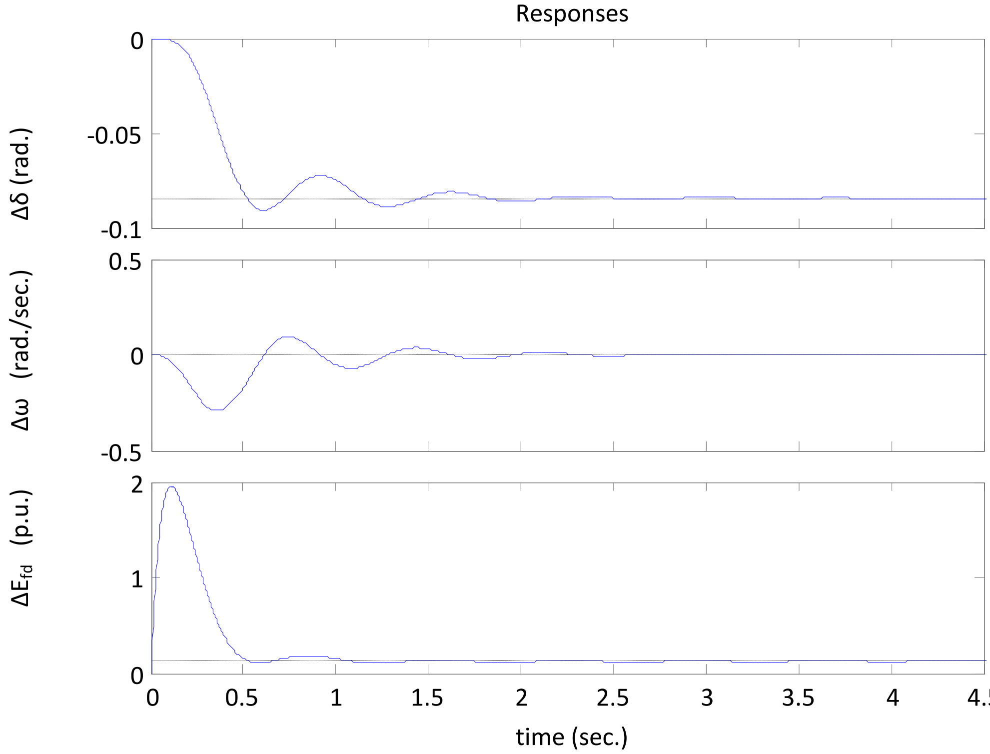


Figure3. *Responses of the output variables of the original continuous open-loop power system model to step input change: ΔVref.=0.05 p.u.*

The computed discrete linear open-loop power system model, based on the associated linearized continuous open-loop system model described in Appendix B of [17], is given below in terms of its matrices with sampling period T_0 = 0.8 sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} 0.1883 & 0.0251 & 0.1037 & -0.0540 & 0.0367 & 0.0016 \\ -2.5406 & 0.1883 & -1.3867 & -0.5211 & -2.2187 & 0.0089 \\ 0.0342 & -0.0016 & 0.0252 & 0.0019 & 0.0504 & 0.0003 \\ -0.0217 & -0.0034 & -0.0078 & 0.0052 & 0.0157 & -0.0001 \\ -0.1176 & 0.0010 & -0.0497 & -0.0126 & -0.0412 & -0.0003 \\ -0.0538 & 0.1590 & 0.0333 & -0.3734 & -0.7702 & 0.0041 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = \begin{bmatrix} -1.5541 & 1.6288 & 2.1552 & 1.5042 & 0.8877 & 3.5881 \end{bmatrix}^T$$

$$\mathbf{C}_{ol/d} = \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D}_{ol/d} = \mathbf{B}_{ol/d}$$

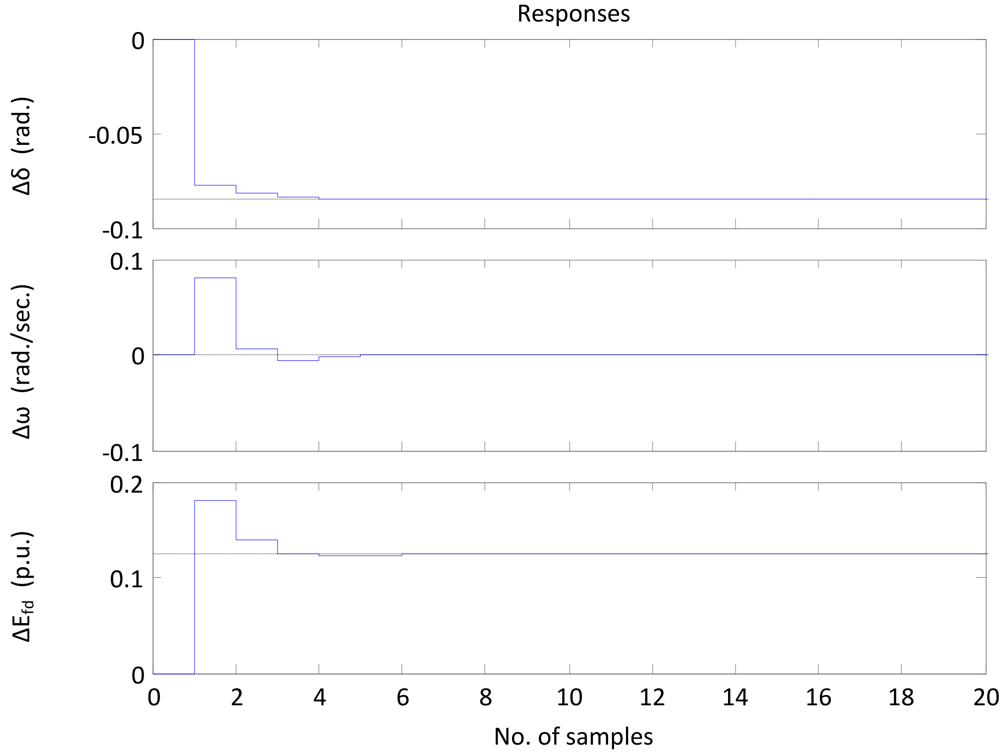


Figure4. *Responses of $\Delta\delta$, $\Delta\omega$, ΔE_{fd} of the output variables of the discrete open-loop system model subject to step change ΔV_{ref} = 0.05 p.u.*

Based on Figure 1, the H^{∞}-control using MROCs, and the computed discrete linear open-loop model of the power system under study, and the discrete closed-loop power system model were designed considering with γ = 0.6 the feedback gain computed as:

L_u =9.3132e-10

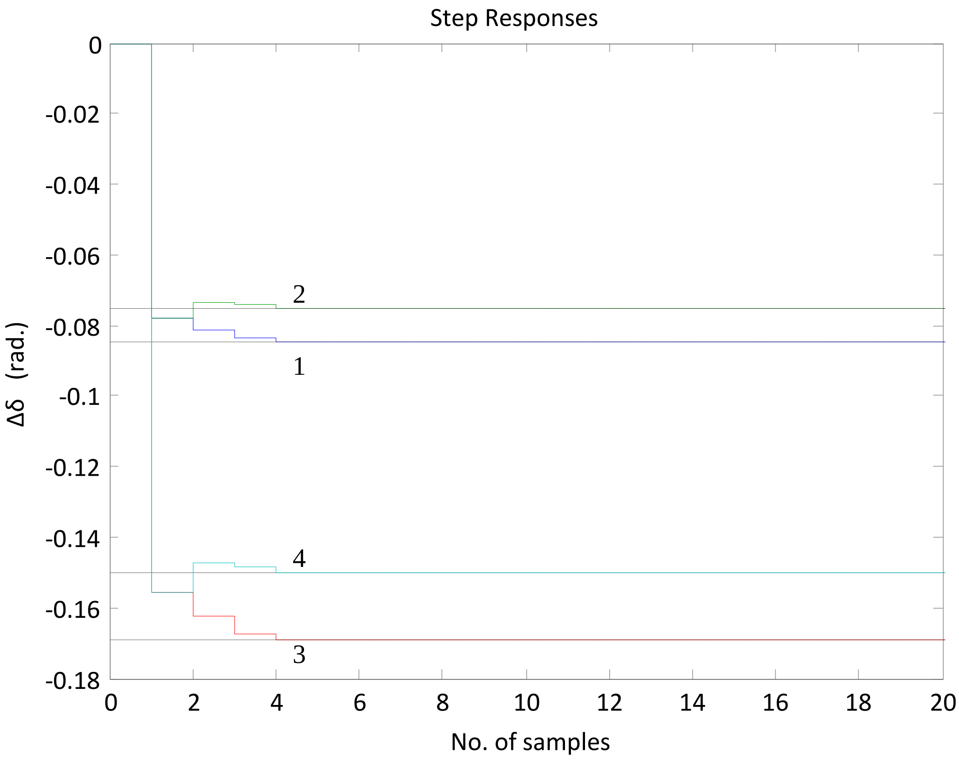
L_γ = [0.0756 -0.0178 0.0346 0.0450 0.1162 -0.0001]

The magnitude of the eigenvalues of the discrete original open-loop and designed closed-loop power system models are shown in Table 2. By comparing the eigenvalues of the designed closed-loop power system models to those of the original open-loop power system model the resulting enhancement in dynamic system stability is judged as being remarkable.

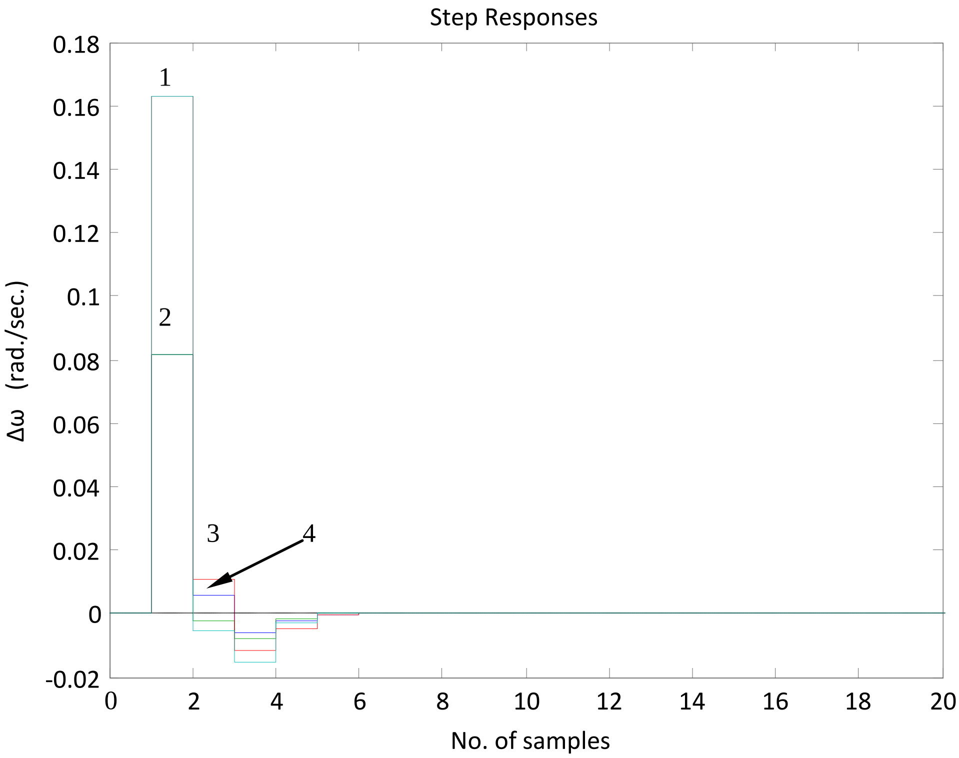
Table2. *Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.*

Original open-loop power system model	$\lvert\lambda\rvert$	0.2834	0.2834	0.0534	0.0020	0.0020	0
Designed closed-loop power system model	$\lvert\hat{\lambda}\rvert$	0.2777	0.2777	0.0285	0.0285	0	0

The responses of the output variables ($\Delta\delta$, $\Delta\omega$, ΔE_{fd}) of the open-loop and the designed closed-loop power system models for zero initial conditions and unit step input disturbance are shown in Figure 5. respectively.



(A)



(B)

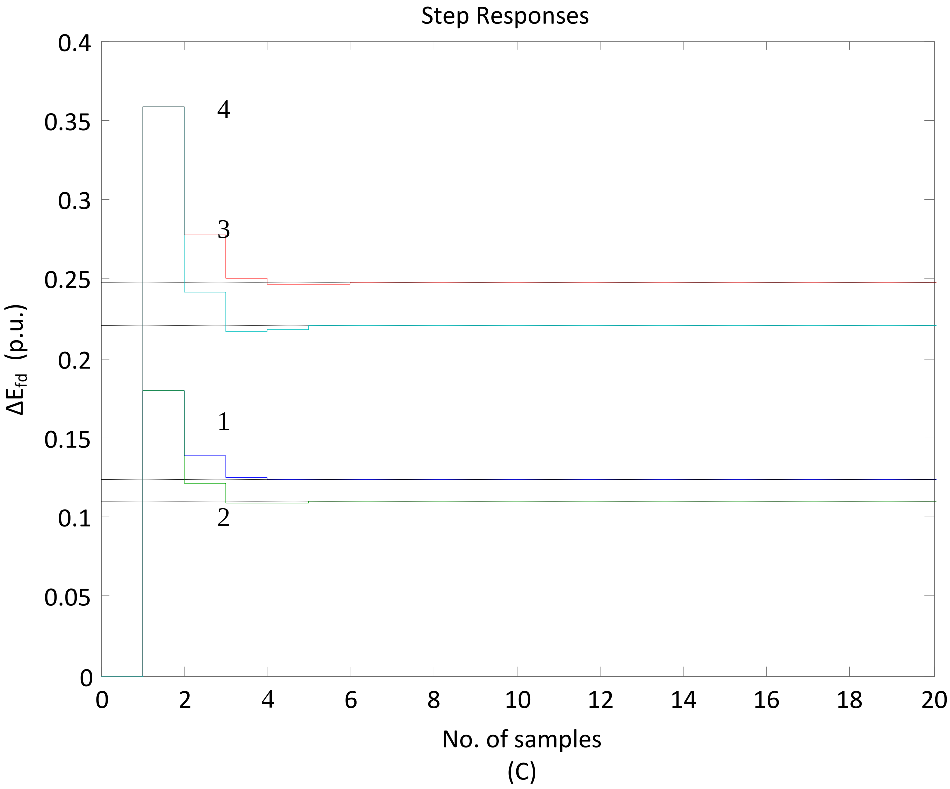


Figure5. *(A), (B), (C): Simulation results represent the responses of the discrete control system:*
(1), (3) for the open loop to step input power load changes $\Delta V_{ref.}$ =0.05 p.u. and 0.10 p.u. respectively.
(2), (4) for the closed loop to step input power load changes $\Delta V_{ref.}$=0.05 p.u. and 0.10 p.u. respectively

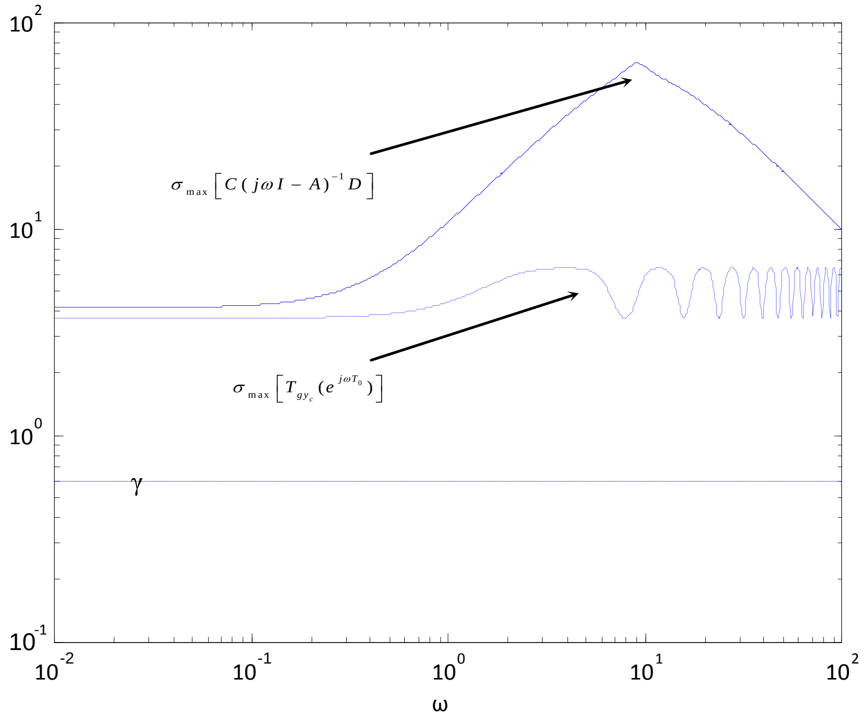


Figure6. *The maximum singular value of $T_{qyc}(z)$ over ω, for the o.p. #1*

From Figure 5 it is clear that the dynamic stability characteristics of the designed discrete closed-loop system-models are far more superior than the corresponding ones of the original open-loop model, which attests in favour of the proposed H^{∞}-control technique.

It is to be noted that the solution results of the discrete system models, i.e. eigenvalues, eigenvectors, responses of system variables etc., for zero initial conditions were obtained by special m-files which were build, customized and executed in MATLAB®.

In Figure 6, the maximum singular value of $T_{qyc}(z)$ is depicted, as a function of the frequency ω. Clearly, the design requirement $\left\| T_{qy_c}(z)_\infty \right\| \le 0.6$, is satisfied. Moreover, as it can be easily checked the poles of the closed loop system, lie inside the unit circle. Therefore, the requirement for the stability of the closed-loop system is also satisfied.

Not that, the H^∞-norm of the open-loop system transfer function between disturbances and controlled outputs has the value `$\left\| C(j\omega I - A)^{-1} B \right\|_\infty = 63.19$.

CONCLUSIONS

The method, H∞ -control based on MROCs was applied successfully to a discrete open-loop power system model (which was computed from an original continuous linearized open-loop one) resulting in the design of an associated discrete closed-loop power system model. The results of the simulations performed on the discrete open- and closed-loop power system models demonstrated clearly the significant enhancement of the dynamic stability characteristics achieved by the designed closed-loop model. Thus this H∞ -control technique was proved to be a reliable tool for the design of implementable MROCs.

REFERENCES

[1] Mats F. Sagfors, H. T. Toivonem and B. Lennartsons. *“H^∞ control of multirate sampled-data systems: A state-space approach”*. Automatica, Vol. 34, 1998, pp. 415-428.

[2] A. A. Stoorvogel. *“The discrete time H^∞-control problem with measurement feedback”*. SIAM J. Control Optim., Vol. 30, 1992, pp. 180-202.

[3] P. Iglesias and K. Glover. *“State-space approach to discrete-time H^∞-control”*. Int. J. Control, Vol. 54, 1991, pp. 1031-1074.

[4] A. A. Stoorvogel. “*The* H^∞-C*ontrol Problem”.* Prentice Hall, Englewood Cliffs, New Jersey, 1992.

[5] J. C, Doyle, K. Glover, P. P. Khargonekar and B. A. Francis. *“State space solutions to standard H^2 and H^∞-control problems”*. IEEE Trans. Autom. Control, Vol. AC-34, 1989, pp. 831-847.

[6] I. Yaesh and U. Shaked. *“Minimum H^∞-norm regulation of linear discrete-time systems and its relation to linear quadratic discrete games”*. IEEE Trans. Autom. Control, Vol. AC-35, 1990, pp. 1061-1064.

[7] Chen and L. Qiu. *“H^∞ design of general multirate sampled-data control systems”*. Automatica, Vol. 30, 1994, pp. 1139-1152.

[8] E. Prempain and I. Postlethwaite. *“Static H^∞ loop shaping control of a fly-by-wire helicopter”*. Automatica, Vol. 41(9), 2005, pp. 1517-1528.

[9] F. Yang, Z. Wang, Y. S. Hung, et al. *“H^∞ control for networked systems with random communication delays”*, IEEE Trans. On Automatic Control, Vol 51(3), 2006, pp. 511-518.

[10] B. Yu, Y. Shi and Huang. *“l-2-l-infinity filtering for multirate systems based on lifted models”*. Circuits Syst. Signal Process, Vol. 27(5), 2008, pp. 699-711.

[11] W. A. Zhang, L. Yu and S. Yin. *“A switched system approach to H^∞ control of networked control systems with time-varying delays”*. Journal of the Franklin Institute, Vol. 348, No. 2, 2011, pp. 165-178.

[12] Z. N. Gao, R.H. Xie, W.H. Fan and Q.W. Chen. *“H^∞ control of wireless networked control systems with signal difference—based deadband scheduling”*. Control and Decision, Vol. 27, No. 9, 2012, pp. 1001-1307.

[13] E. G. Tian and D. Yue. *“Reliable H^∞ filter design for T-S fuzzy model-based networked control systems with random sensor failure”*. International Journal of Robust and Nonlinear Control, Vol. 23, No. 1, 2013, pp. 15-32.

[14] A. Isidori and A. Astolfi. *“Disturbance attenuation and H^∞-control via measurement feedback in nonlinear systems”*. Journal: IEEE Transactions on Automatic Control-Vol. 37, No. 9, 1992, pp. 1283-1293.

[15] K. G. Arvanitis and P. N. Paraskevopoulos. *"Sampled-data minimum H^{∞}-norm regulation of linear continuous-time systems using multirate-output controllers"*. J. Optim.Theory Appls., Vol. 87, 1995, pp. 235-267.

[16] A. K. Boglou, D. I. Pappas and D. N. Sapountzis. *"Design of Discrete Optimal Multirate-Output Controllers Applied to a Hydrogenerator Power System"*, American Journal of Engineering Research (AJER), Vol. 3, Issue 2, 2014, pp. 170-183.

[17] E. J. Davison and N. S. Rau. *"The optimal output feedback control of a synchronous machine"*. IEEE Trans. Power App. Syst., PAS-90, 1971,pp. 2123-2134.

[18] J. R. Smith, D.P. Papadopoulos, C.J. Cudworth and J. Penman. *"Prediction of Forces on the Retaining Structure of hydrogenerator Diring Severe Disturbance Contitions"*. Electric Power Systems Research, Vol 14, 1988, pp. 1-9.

[19] A. K. Boglou and D. P. Papadopoulos. *"Dynamic Performance Improvement of Hydrogenerator with Modern Pole-Assignment Control Methods"*. J. Electrical Engineering, 46, No. 3, 1995, pp. 81-89.

[20] R. H. Park. *"Two reaction theory of synchronous machines: General method of analysis"*. Part I. Trans. AIEE, Vol. 48, 716, 1929.

APPENDIX A

Notation

i	instantaneous value of the current
v	instantaneous value of the voltage
v_o	infinite bus voltage
ψ	flux-linkage
R_a	stator resistance
R	resistance
x	reactance
δ	torque angle, in rad.
ω	generator speed (ω_o=synchronous speed) rad/sec.
H	inertia constant in sec.
i_t	generator terminal current
v_t	generator terminal voltage
E_{fd}	exciter output voltage
V_{ref}.	Reference voltage
P_t, Q_t	active and reactive power delivered at generator terminals
K_e	exciter amplifier gain
τ_e	exciter amplifier constant, in sec.
x_{ad}, x_{aq}	magnetizing reactance in d- and q-axis, respectively
x_{ld}, x_{lq}	linkage reactances in d- and q-axis, respectively
R_T, X_T	transformer resistance and reactance
R_L, X_L	transmission line resistance and reactance
R_e, X_e	external system resistance and reactance
Δ	linearized quantity
o.p.	operating point

Subscripts

d, q direct-and quadrature-axis quantities

f field winding quantities

D, Q d- and q-axis damper quantities 3.

Numerical Values of the System Parameters (p.u. values on generator ratings, the time constants and the inertia constant of the generator and prime-mover are in sec.)**.**

Principal System Data

MVA = 117	x_f = 0.221	**AVR and exciter**
kV = 15.75	x_D = 0.992	K_e =50.0
RPM = 125	x_Q = 0.551	τ_e = 0.05
x_d = 0.935	R_f = 0.0006	**Transformer and**
x_q = 0.574	R_D = 0.014	**double-circuit**
x_{ad} = 0.827	R_Q = 0.008	**transmission line:**
x_{1d} = 0.095	R_a = 0.002	R_e =0.015
x_{aq} =0.475	H = 3.0 in sec	X_e = 0.40
x_{1q} = 0.076		

Table3. *The operating points (o.p.) of the hydrogenerator system selected in this study*

	v_t (p.u.)	P_t (p.u.)	Q_t (p.u.)
o.p.	1.1	0.4	-0.68

APPENDIX B

Numerical values of matrices **A**, **B, C** and **D** of the original continuous 6th-order system

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -103.7339 & 0 & -15.9242 & -37.0447 & 0 & 0 \\ -0.0524 & 0 & -0.6412 & 0.4926 & -90.2318 & 0.2279 \\ -2.8415 & 0 & 11.4934 & -19.5599 & -0.0024 & 0 \\ -4.3876 & 0 & -0.0319 & -0.0740 & 0.1297 & 0 \\ 46.2648 & 0 & -183.6240 & -427.1675 & 209.5864 & -20 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1000 \end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

D=B.