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Propagation of Natural Waves in Extended Cylindrical Viscoelastic Panels

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ABSTRACT

Considered the propagation of harmonic waves in cylindrical panels with variable thickness. To derive the equations of the used Virtual work. Solving boundary value problem obtained by the method orthogonal pivotal condensation Godunov. Dispersion curves were investigated depending on several of geometrical parameters of the system.

Keywords: *cylindrical shell, the hypothesis of the Kirchhoff - Love, harmonic waves, viscoelastic panels, mid-surface.*

INTRODUCTION

Wave processes in the form of elastic fibers in the isotropic and anisotropic cylindrical shells of constant thickness are well studied [1, 2, 3]. A large number of works devoted to the dynamics of shells described by Timoshenko model [4, 5, 6, 7]. In [8] for the study of wave processes used asymptotic methods of wave propagation in a cylindrical shell with a small change in its thickness along the axis. Despite the large number of papers devoted to the problem of wave propagation in waveguides.

The research problem of wave propagation in viscoelastic (cylindrical panel) variable thickness represents a significant theoretical practical interest.

STATEMENT OF THE WAVE PROBLEM

Regarded an endless a deformed a cylindrical panel with a thickness h, densities ρ. In the orthogonal curvilinear coordinate system, $(\alpha_1;\alpha_2;z)$ at $z=0$ shell occupies the region

$$-\infty < \alpha_1 < +\infty;\ 0 < \alpha_2 < l;$$
$$-\frac{h}{2} < z < \frac{h}{2}$$

Curvature of the middle surface $z=0$ are equal $k_1 = 0;\ k_2 = \frac{1}{R}$ respectively coordinates α_1 *and* α_2. Within the framework of hypotheses Kirchhoff - Love the variation component of the displacement vector $u_1(x), u_2(x), u_3(x)$ panels are determined by the following relations [1, 2]

$$u_1(x) = u - \theta_1 z;\quad u_2(x) = \vartheta - \theta_2 z;\quad u_3(x) = w, \tag{1}$$

Where $u, \vartheta, w-$ components of the displacement vector of the middle surface; θ_1, θ_2- rotation angles with respect to the normal axis α_1 *and* α_2.

To derive the equations panel used Virtual work

$$\delta\Pi = \delta T \tag{2}$$

Where $\delta\Pi$ – variation of the potential energy of the shell; δT – virtual work of the inertial forces panels mass. In this paper V.V. Novozhilov [1], taking into account the relations (1) the deduction for the following expression based on the linear theory of elasticity

$$\delta\Pi = \int_F \{T_1\delta\varepsilon_1 + T_2\delta\varepsilon_2 + S\delta\varepsilon_{12} + M_1\delta\chi_1 + M_2\delta\chi_2 + 2N\delta\tau\} d\alpha_1 d\alpha_2, \tag{3}$$

where $T_1, T_2, S, M_1, M_2, N-$ forces and moments; $\varepsilon_1, \varepsilon_2, \varepsilon_{12}, \chi_1, \chi_2, \tau$ – components of deformation of the middle surface. In (3) we have omitted terms are of the order $\frac{h}{R}$.

According to [1] the tangential components of bending deformation of the middle surface are expressed in terms of its movement and rotation angles are normal as follows

$$\varepsilon_1 = \frac{\partial u}{\partial \alpha_1}; \varepsilon_2 = \frac{\partial \vartheta}{\partial \alpha_2} + k_2 w; \varepsilon_{12} = \frac{\partial \vartheta}{\partial \alpha_1} + \frac{\partial u}{\partial \alpha_2};$$
$$\chi_1 = \frac{\partial \theta_1}{\partial \alpha_1}; \quad \chi_2 = \frac{\partial \theta_2}{\partial \alpha_2}; \quad \tau = \frac{\partial \theta_2}{\partial \alpha_1}; \tag{4}$$
$$\theta_1 = -\frac{\partial w}{\partial \alpha_1}; \quad \theta_2 = -\frac{\partial w}{\partial \alpha_2} + k_2 \vartheta$$

In turn, forces and moments associated with the components of the strain of defining relations arising from the generalized Hooke's law:

$$T_1 = \tilde{c}(\varepsilon_1 + \nu\varepsilon_2),$$
$$M_1 = \tilde{D}(\chi_1 - \nu\chi_2),\ S = \tilde{A}\varepsilon_{12}; N = \tilde{B}\tau$$

where

$$\tilde{c} = \frac{\tilde{E}h}{1-\nu^2}; \quad \tilde{D} = \frac{\tilde{E}h^3}{12(1-\nu^2)};$$
$$\tilde{A} = \frac{\tilde{E}h}{2(1+\nu)}; \quad \tilde{B} = \frac{\tilde{E}h^3}{12(1+\nu)}$$

$\tilde{E}$ – modulus operator, which have the form [9].:

$$\tilde{E}\varphi(t) = E_{01}\left[\varphi(t) - \int_0^t R_E(t-\tau)\varphi(t)d\tau\right] \tag{5}$$

$\varphi(t)$ – Arbitrary function of time; $R_E(t-\tau)$ – relaxation kernel; E_{01} – instantaneous modulus of elasticity; Accept the integral terms in (5) small, then the function $\varphi(t) = \psi(t)e^{-i\omega_R t}$, where $\psi(t)$- slowly varying function of time, ω_R - real constant. Next, using the procedure of freezing [9], we note the relation (5) approximate species

$$\bar{E}\varphi = E\left[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R)\right]\varphi = \bar{E}\varphi,$$

where

$$\Gamma^C(\omega_R) = \int_0^\infty R(\tau)\cos\omega_R\tau\, d\tau,$$

$\Gamma^C(\omega_R) = \int_0^\infty R(\tau)\cos\omega_R\tau\, d\tau$, respectively, the cosine and sine Fourier transforms core material relaxation. As an example, assume three viscoelastic relaxation parametric kernel $R(t) = Ae^{-\beta t}/t^{1-\alpha}$, has a weak singularity[9]. ν – Poisson's ratio. It is supposed that the inertial forces in the corners θ_1 and θ_2 small and compared to the other forces of inertia. Given this, if we neglect the inertia of the normal rotation, the virtual work of the inertial forces shell can be written as:

$$\delta T == -\int_F \rho h(\ddot{u}\delta u + \ddot{\vartheta}\delta\vartheta + \ddot{w}\delta w)d\alpha_1 d\alpha_2 \tag{6}$$

After substituting (3) and (6) in (2) and standard procedures for integration by parts, taking into account the relation (4) we obtain the equations of motion in the form of

$$\frac{\partial T_1}{\partial \alpha_1} + \frac{\partial S}{\partial \alpha_2} = -\rho h \frac{\partial^2 u}{\partial t^2} \tag{7}$$
$$\frac{\partial T_2}{\partial \alpha_2} + \frac{\partial S}{\partial \alpha_1} + k_2 Q_2 = -\rho h \frac{\partial^2 \vartheta}{\partial t^2}$$
$$\frac{\partial Q_1}{\partial \alpha_1} + \frac{\partial Q_2}{\partial \alpha_2} - k_2 T_2 = -\rho h \frac{\partial^2 w}{\partial t^2}$$

$$Q_1 = \frac{\partial M_1}{\partial \alpha_1}; \quad Q_2 = \frac{\partial M_2}{\partial \alpha_2} + 2\frac{\partial N}{\partial \alpha_1} \tag{8}$$

Alternative boundary conditions of the free edge, or anchorage, with $\alpha_2 = 0, l$ are the following: the free edge

$$S = 0; \quad T_2 = 0; \quad M_1 = 0; \quad Q_2 = 0$$

(9)

rigid seal

$$u = 0, \quad \vartheta = 0, \quad w = 0, \quad Q_2 = 0 \tag{10}$$

Using relationships (4) and (5), (7), (8) a complete system of equations of motion can be written as eight differential equations Placing on the first derivatives α_2:

$$\bar{A}\frac{\partial u}{\partial \alpha_2} = S - \bar{A}\frac{\partial \vartheta}{\partial \alpha_1}; \qquad ;$$
$$\bar{c}\frac{\partial \vartheta}{\partial \alpha_2} = T_2 - \bar{c}\nu\frac{\partial u}{\partial \alpha_1} - \bar{c}k_2 w$$
$$\bar{D}\frac{\partial \theta_2}{\partial \alpha_2} = M_2 + \frac{\partial^2 w}{\partial \alpha_1^2};$$
$$\frac{\partial w}{\partial \alpha_2} = -\theta_2 + k_2\vartheta$$
$$\frac{\partial S}{\partial \alpha_2} = -\rho h\frac{\partial^2 u}{\partial t^2} - \bar{c}\frac{\partial^2 u}{\partial \alpha_1^2} - \nu\frac{\partial T_2}{\partial \alpha_1}$$
$$\frac{\partial T_2}{\partial \alpha_2} = -\rho h\frac{\partial^2 \vartheta}{\partial t^2} - \frac{\partial S}{\partial \alpha_1} - k_2 Q_2 \tag{11}$$
$$\frac{\partial Q_2}{\partial \alpha_2} = -\rho h\frac{\partial^2 w}{\partial t^2} + \bar{D}\frac{\partial^4 w}{\partial \alpha_1^4} - \nu\frac{\partial^2 M_2}{\partial \alpha_1^2} + k_2 T_2;$$
$$\frac{\partial M_2}{\partial \alpha_2} = Q_2 - 2\bar{B}\frac{\partial^2 \theta_2}{\partial \alpha_1^2},$$

where

$$\bar{c} = \frac{\bar{E}h}{1-\nu^2}; \quad \bar{D} = \frac{\bar{E}h^3}{12(1-\nu^2)};$$
$$\bar{A} = \frac{\bar{E}h}{2(1+\nu)}; \quad \bar{B} = \frac{\bar{E}h^3}{12(1+\nu)}$$

In the case of traveling along α_1 harmonic wave solutions of the boundary value problem for the system (11) with boundary conditions of (9) and (10) allow separation of variables.

$$u = z_1 e^{i(k\alpha_1 - \omega t)};$$

$$\vartheta = z_2 e^{i(k\alpha_1 - \omega t)}; \tag{12}$$

$$w = z_3 e^{i(k\alpha_1 - \omega t)};$$

$$\theta_2 = z_4 e^{i(k\alpha_1 - \omega t)};$$

$$S = z_5 e^{i(k\alpha_1 - \omega t)};$$

$$T_2 = z_6 e^{i(k\alpha_1 - \omega t)};$$

$$\theta_2 = z_7 e^{i(k\alpha_1 - \omega t)};$$

$$M_2 = z_8 e^{i(k\alpha_1 - \omega t)};$$

where $\omega = \omega_R + i\omega_I$ – complex natural frequency; k – the wave number; ω_R – the real part of the complex frequency; ρ – density; $z_j(\alpha_2)(j = 1,2,3..8)$ – function waveform. To ascertain their physical meaning of the case:

1) $k = \kappa_R$; $V = C_R + iC_I$ – Then the solution of (9) is given by a sine wave z, whose amplitude decays over time;

2) $k = \kappa_R + i\kappa_I$; $V = C_R$ – Then at each point z fluctuations established by α_1 damped.

Further assuming that both the shell edge $\alpha_2 = 0$ and $\alpha_2 = l$ – free. After substitution of (12) in equation (11) and taking into account the boundary conditions (9), we have the spectral Boundary Value Problem ω for a system of eight ordinary differential equations for the complex function form:

$$z_1' = z_5/\bar{A} + kz_2, z_2' = z_6/\bar{c} + \nu k z_1 - k_2 z_3,$$
$$z_3' = -z_4 + k_2 z_2, z_4' = z_8/D - \nu k^2 z_3,$$
$$z_5' = h(\bar{E}k^2 - \rho\omega^2)z_1 + \nu h^2 z_6, \tag{13}$$
$$z_6' = -h\rho\omega^2 z_2 - kz_5 - k_2 z_7,$$
$$z_7' = -h\rho\omega^2 z_3 + \bar{E}/12h^3k^4 z_3 + \nu k^2 z_8 + k_2 z_6;$$
$$z_8' = z_7 + \bar{G}/3h^3k^2 z_4; \quad z_5 = z_6 = z_7 = z_8 = 0;$$
$$\alpha_2 = 0, l$$

In the analysis of the dispersion parameter of harmonic waves k assume given.

NUMERICAL ANALYSIS OF THE DISPERSION OF NORMAL WAVES IN CYLINDRICAL PANELS

Based on the solution of the problem (13) orthogonal sweep method of Godunov was performed numerical analysis of the dispersion of these waves.

Fig. 1 and 2 shows the dependence of the real part of the complex phase velocity of the first two modes of the wave number for various waveguides. In all variants of calculation, the following dimensionless parameters canister

$$E = 1, \rho = 1, \nu = 0{,}25, l = 1,$$
$$A = 0{,}048; \quad \beta = 0{,}05; \quad \alpha = 0{,}1.$$

Thickness h varies linearly

$$h(\alpha_2) = h_1 + \Delta h\ \alpha_2 \tag{14}$$

$$\Delta h = (h_2 - h_1)/l$$

The solid lines in the figures correspond to the embodiments of the constant thickness $(h_1 = h_2 = 0.1)$, the dotted lines characterize the panel with a tapered section ($\Delta h = 0.0001$). In the latter case, $h_2 = 0.1$, and the thickness $h_1 = 0.001$. Parameters constant k_2 of curvature and takes values of 45^0 and 90^0. The broken line in Fig. 1 and 2 correspond to the considered case of Kirchhoff plates with -Lave $k_2 = 0$. From Figures 1 and 2 show a qualitative difference in the behavior of the dispersion curves of the first mode, the corresponding shell and plate. If in the second phase velocity curve is monotonic in the first case there is a typical maximum range in the medium, which is attributed to higher flexural shell severity as compared with the plate. Actual speed of the second mode, unlike the case of the constant thickness also generally increased with increasing curvature. At the same time, as one would expect, the larger curvature k_2 more slowly takes you to a site without dispersion movement $(c = const)$ with increasing wave number.

As for the localization, it increases with increasing curvature (for sufficiently large k for example, in $k = 10$). Moreover, this increased localization in the cylindrical panel is characteristic for both modes (real part of the complex velocity). With the growth parameter k_2 there is a tendency to increase speed (C_R) flexural mode and reduce the rate of tensional modes. Speed damping coefficient (C_I)

bending mode decreases the rate of the parameters k_2 and increases the rate of decay Corresponds to the torsion mode.

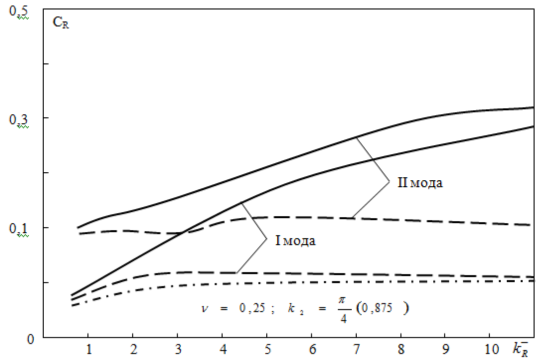

Figure1. *The dependence of the real part of the wave propagation velocity of the wave number*

Conclusions

With the increase in the curvature of the cylindrical constant thickness increases the real part of the complex $\left(C_R = \mathrm{Re}\,al(V)\right)$ the propagation velocity of the first bending mode and decreases the speed of propagation of the second tensional mode.

In the case of a wedge-shaped cylindrical panel for each mode, there are limits propagation velocity with increasing wave number coinciding in magnitude with the corresponding velocities of normal waves in a wedge-shaped plate of zero curvature. In the short-range localization movement exists and increases with the curvature of the panel.

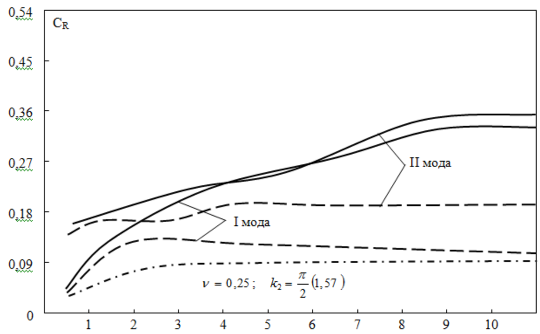

Figure2. *Dependence of the real speed* (C_R) *propagation of the wave number*

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