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A New Test for Multipartite Entanglement in Bell-Type Experiments

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ABSTRACT

In trial, we especially consider inequalities for confirming multipartite entanglement from experimental data obtained in Bell-type experiments. We present new entanglement witness inequalities. Some physical situation is that we measure σx, σy, and σz per side. Our analysis discovers a new multipartite entangled state and it is experimentally feasible. If the reduction factor V of the interferometric contrast observed in a N-particle correlation experiment is V > 0.4, then a measured state is full N-partite entanglement in a significant specific case. It is not revealed by previous Bell type experimentally feasible methods presented in [17], which states if V > 0.5 then the significant specific type state is full N-partite entanglement.

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INTRODUCTION

Since the Svetlichny inequality, it has been a problem how to confirm multipartite entanglement experimentally [1]. And we have been given precious experimental data by efforts of experimentalists [2—6]. Proper analysis of these experimental data then becomes necessary, and as a result of such analysis [7], the experimental data obtained by Pan and co-workers [5] confirms the existence of genuinely three-particle entanglement in 2000. More recently, experimental violation of multipartite Bell inequalities with trapped ions is reported [8]. Device independent tomography of multipartite quantum states is reported [9]. Demonstration of genuine multipartite entanglement with device-independent witnesses is also reported [10].

There have been many researches on the multipartite entanglement problem, providing inequalities for functions of experimental correlations [1, 7, 11—18]. Uffink introduced a nonlinear inequality aimed at giving stronger tests for full N-partite entanglement than previous formulas. It was also discussed that when the two measured observables are assumed to precisely anticommute, a stronger quadratic inequality can be used as a witness of full N-partite entanglement [17].

After that there are many researches of multipartite entanglement (cf. [19, 20]). We do not know that the inequality presented in [17] is the optimal way in detection of multipartite entanglement in Bell-type experiment. In fact it is not so if we introduce measuring σ_z per side. Here, we study more efficient way in this case.

In this paper, we investigate inequalities for confirming multipartite entanglement from experimental data obtained in Bell-type experiments. We present new inequalities to do so. some physical situation is that we measure

σ_x, σ_y, and σ_z per side. Our analysis discovers a new multipartite entangled state and it is experimentally feasible. If the reduction factor V of the interferometric contrast observed in a N-particle correlation experiment is $V > 0.4$, then a measured state is full N-partite entanglement in a significant specific case. It is not revealed by previous Bell-type experimentally feasible methods presented in [17], which states if $V > 0.5$ then the significant specific type state is full N-partite entanglement.

TESTS OF MULTIPARTITE ENTANGLEMENT

We want to know if the following multipartite state is full N-partite entanglement experimentally. The value of V can be interpreted as the reduction factor of the interferometric contrast observed in a N-particle correlation experiment.

$\rho = V |GHZ\rangle\langle GHZ| + (1 - V) |1...1\rangle\langle 1...1|$, (1)

where $|GHZ\rangle = |1...1\rangle + |0...0\rangle /\sqrt{2}$ is the N-partite Greenberger-Horne-Zeilinger (GHZ) state [21].

Lemma

In what follows, we use the following lemma. Lemma [17]: Let $-1 \le A, B \le 1$ be Hermitian operators satisfying $\{A, B\} = 0$. Then

$(A)^2 + (B)^2 \le 1.$ (2)

Proof: Suppose that $\{A,B\} = 0$ and $-1 \le A,B \le 1$. Let us take $C = A \cos\theta + B \sin\theta$, and derive the maximum value of $tr[\rho C]$. Since we are interested only in the maximum, we may assume $A^2 = B^2 = 1$. Then we get $C^2 = 1 + (1/2)\{A, B\} \sin 2\theta = 1$. The variance inequality leads to $|tr[\rho C]|^2 \le tr[\rho C^2] = 1$. Now take $\cos\theta = (A) /\sqrt{(A)^2 + (B)^2}$, $\sin\theta = (B) /\sqrt{(A)^2 + (B)^2}$, then we get $(A)^2 + (B)^2 \le 1$. QED.

Reviews

In what follows, we review previous methods. We cannot see if the multipartite state (1) is fully entangled when $V < 0.5$.

Let us consider the following Bell operators [22, 23]

$X_N = 2^{(N-1)/2}(|1...1\rangle\langle 0...0| + |0...0\rangle\langle 1...1|)$,

$Y_N = 2^{(N-1)/2}(-i|1...1\rangle\langle 0...0| + i|0...0\rangle\langle 1...1|)$. (3)

We can measure the following operators by Bell-type experiments measuring σ_x and σ_y per side:

$X = (2) (|1...1\rangle\langle 0...0| + |0...0\rangle\langle 1...1|), Y = (2) (-i|1...1\rangle\langle 0...0| + i|0...0\rangle\langle 1...1|)$. (4)

We may assume $-1 \le X, Y \le 1$ when the system is not in full N-partite entanglement. In fact, we have the following entanglement witness inequalities [18]

$|(X)| \le 1, |(Y)| \le 1.$ (5)

A violation of the above relations (5) means full N-partite entanglement. Let us consider the quantum state (1). After some algebra, we find that

$|(X)| = 2V, |(Y)| = 0.$ (6)

Hence we cannot see if the multipartite state (1) is fully entangled when we only use the formulas (5) and

$V \le 1/2.$ (7)

From Lemma described above, we have the following entanglement witness inequality because $\{X, Y\} = 0$ and $-1 \le X, Y \le 1$ [17].

$(X)^2 + (Y)^2 \le 1.$ (8)

A violation of the reration (8) means full N-partite entanglement. Let us consider the quantum state (1). After some algebra, we find that

$(X)^2 + (Y)^2 = (2V)^2.$ (9)

Hence we cannot see if the multipartite state (1) is fully entangled when we only use the formula (8) and

$V \le 1/2.$ (10)

New Method

In what follows, we propose a new methods. We can see if the multipartite state (1) is fully entangled when $0.4 \le V < 0.5$.

Let us consider the following operator.

$Z_N = 2^{(N-1)/2}(|1...1\rangle\langle 1...1| - |0...0\rangle\langle 0...0|)$. (11)

We can measure the following operators by an experiment measuring σz and $I(= +1)$ per side:

$Z = (|1...1\rangle\langle 1...1| - |0...0\rangle\langle 0...0|)$. (12)

Clearly, we see $-1 \le Z \le 1$. Originally, we have the following entanglement witness inequalities [18]

$|(X)| \le 1, |(Y)| \le 1.$ (13)

We have the following quantum inequality for all states

$|\langle Z\rangle| \leq 1$. (14)

We see the following anti-commutation:

$\{X, Y\} = 0$,

$\{Y, Z\} = 0$,

$\{Z, X\} = 0$. (15)

Now we can use the Lemma. From Lemma, we derive a set of quadratic entanglement witness inequalities

$\langle X\rangle^2 + \langle Y\rangle^2 \leq 1$,

$\langle Y\rangle^2 + \langle Z\rangle^2 \leq 1$,

$\langle Z\rangle^2 + \langle X\rangle^2 \leq 1$. (16)

A violation of one of the inequalities (16) implies full N-partite entanglement. We see the following quadratic entanglement witness inequality is not new.

$\langle X\rangle^2 + \langle Y\rangle^2 \leq 1$. (17)

We see the following quadratic entanglement witness inequalities are new.

$\langle Y\rangle^2 + \langle Z\rangle^2 \leq 1$,

$\langle Z\rangle^2 + \langle X\rangle^2 \leq 1$. (18)

In what follows, we use the following new entanglement witness inequality:

$\langle Z\rangle^2 + \langle X\rangle^2 \leq 1$. (19)

Let us consider the quantum state (1). We get the following from the GHZ state

$\langle X\rangle^2 = (2V)^2$ (20)

We get the following from the colored noise state.

$\langle Z\rangle^2 = (1 - V)^2$ (21)

Thus we find that

$\langle X\rangle^2 + \langle Z\rangle^2 = (2V)^2 + (1 - V)^2$. (22)

Hence we can see that the multipartite state (1) is fully entangled when

$(2V)^2 + (1 - V)^2 > 1$. (23)

For example, if $V = 1/2$ then

$(2V)^2 + (1 - V)^2 = 1 + 1/4 > 1$. (24)

Thus, the multipartite state (1) is fully entangled. It is not revealed by previous Bell-type experimentally feasible methods presented in [17]. In fact, we see

$(2V)^2 + (1 - V)^2 = 5V^2 - 2V + 1$. (25)

Thus, if $5V^2 - 2V > 0$ that is $V > 2/5 = 0.4$, then the multipartite state (1) is fully entangled. Therefore we present a new method of detecting full N-partite entanglement. Are there more efficient ways? This is open.

Conclusions

In conclusions, we have considered inequalities for confirming multipartite entanglement from experimental data obtained in Bell-type experiments. We have presented new entanglement witness inequalities. Some physical situation has been that we measure σ_x, σ_y, and σ_z per side. Our analysis has discovered a new multipartite entangled state and it has been experimentally feasible. If the reduction factor V of the interferometric contrast observed in a N-particle correlation experiment has been $V > 0.4$, then a measured state has been full N-partite entanglement in a significant specific case. It has not been revealed by previous Bell-type experimentally feasible methods presented in [17], which states if $V > 0.5$ then the significant specific type state is full N-partite entanglement.

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