International Journal of Emerging Engineering Research and Technology
Volume 5, Issue 9, 2017, PP 37-46
ISSN 2349-4395 (Print) & ISSN 2349-4409 (Online)

Intercomparison of Estimates of Maximum Flood Discharge Using Quantitative and Qualitative Assessments Adopting L-Moments in Parameter Estimation of Probability Distributions

N. Vivekanandan[1], C. Ramesh[2]

[1]*Scientist-B, Central Water and Power Research Station, Pune, Maharashtra, India*
[2]*Scientist-C, Central Water and Power Research Station, Pune, Maharashtra, India*

***Corresponding Author:** N. Vivekanandan, Scientist-B, Central Water and Power Research Station, Pune 411024, Maharashtra, India*

ABSTRACT

Estimation of Maximum Flood Discharge (MFD) at a desired location on a river is important for planning, design and management of hydraulic structures. This can be achieved using deterministic models with extreme storm events or through frequency analysis by fitting of probability distributions to the observed Annual Maximum Discharge (AMD) data. In the latter approach, suitable probability distributions and associated parameter estimation methods are applied. In this paper, Method of Moments and L-Moments (LMO) are used for determination of parameters of five probability distributions. The adequacy of fitting of probability distributions to the observed data is quantitatively assessed by applying Goodness-of-Fit (GoF) tests i.e., Chi-square and Kolmogorov-Smirnov. In addition to GoF tests, model performance indicators such as correlation coefficient and mean absolute percentage error are used to assess the performance of the probability distribution models. However, the results of GoF tests offered diverging inferences which lead to adopt qualitative assessment using probability plots to aid the selection of suitable distribution for estimation of MFD. The analysis of the outcomes of MFD thus inferred Generalized Extreme Value (LMO) distribution as better suited amongst five selected probability distributions studied for estimation of MFD for river Ganga at Allahabad and Varanasi gauging sites.

Keywords: *Correlation coefficient, Chi-square test, Flood frequency analysis, Kolmogorov-Smirnov test, Maximum flood discharge, Mean absolute percentage error, Probability distribution*

INTRODUCTION

Estimation of Maximum Flood Discharge (MFD) with certain return period of occurrence are important for the design of hydraulic structures such as bridges, barrages, culverts, dams and flood protection works. Since the phenomenon of relevance of the MFD is highly stochastic in nature, the MFD can be effectively determined by fitting of probability distributions to the series of observed Annual Maximum Discharge (AMD) data [1]. An AMD is the highest instantaneous discharge value at a definite cross-section of a natural stream/ river as an entire year. The longer the period of observation would offer a longer length of the series which could offer better results in Flood Frequency Analysis (FFA).

The probability distributions that are adopted in FFA include Exponential (EXP), Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2), Generalized Extreme Value (GEV) and Generalized Pareto (GP) distributions [2]. Generally, Method of Moments (MoM) is used in determining the parameters of the probability distributions. Sometimes, it is difficult to assess exact information about the shape of a distribution that is conveyed by its third and higher order moments. Also, when the sample size is small, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn. It is also reported that the estimated parameters of distributions fitted using MoM are often less accurate than those obtained by other parameter estimation procedures such as maximum likelihood method, method of least squares and probability weighted moments [3]. To address these shortcomings, the application of alternative approach [4], L-Moments (LMO) discussed in this paper is used for FFA. Kjeldsen et al. [5] applied LMO in Regional FFA (RFFA) for KwaZulu-Natal Province of South Africa.

Kumar et al. [6] carried out RFFA adopting twelve frequency distributions using LMO and found that the GEV is better suited distribution for eight gauging sites. Yue and Wang [7] applied LMO to identify the suitable probability distribution for modelling of annual stream flow in different climatic regions of Canada. Kumar and Chatterjee [8] employed the LMO to define homogenous regions within 13 gauging sites in Brahmaputra, India. Atiem and Harmancioglu [9] carried out RFFA adopting GP, GEV, Generalized Logistic (GLO), Generalized Normal (GNO) and Pearson Type-3 (PR3) distributions using LMO for 14 gauged sites on the tributaries of river Nile and found that the GLO is better suited distribution for estimation of peak flows. Abida and Ellouze [10] carried out RFFA adopting GEV, PR3, GLO, GNO and GP distributions using LMO for different zones in Tunisia. They concluded that the GNO distribution is better suited for Northern Tunisia whereas the GNO and GEV distributions for central/ southern Tunisia. Study by Saf [11] revealed that the PR3 distribution is better suited for modelling of extreme values in Antalya and Lower-West Mediterranean sub-regions whereas the Generalized Logistic distribution for the Upper-West Mediterranean sub-region. Bhuyan et al. [12] applied generalized version of LMO (LH-moments) for RFFA of river Brahmaputra. They have found the RFFA based on the GEV distribution by using level one LH-moment (L_1) give better results over LMO. It was reported by Malekinezhad et al. [13] that GEV (using LMO) is better suited for modelling AMD of three different regions in Iran.

Badreldin and Feng [14] carried out the RFFA for the Luanhe basin using LMO and cluster techniques. Hailegeorgis et al. [15] obtained that a two-parameter Gumbel (also referred as EV1) distribution, which has no shape parameter, lacks robustness and hence 'misspecification' of the distribution largely affects quantile estimation of extreme precipitation events for Trondheim City located in mid-Norway. Haberlandt and Radtke [16] carried out FFA using AMD data for three mesoscale catchments in northern Germany. Markiewicz et al. [17] adopted Generalized Exponential (GE) and inverse Gaussian distributions (using LMO) in frequency analysis of annual maximum flows for Polish rivers. They described that the GE occupies as front runner among all distributions commonly used for FFA of Polish data and can be included into the group of the alternative distributions. Kossi et al. [18] carried out RFFA for Volta River Basin (VRB) using LMO of five probability distributions. By using LMO diagrams and GoF test (i.e., Z-statistic), they found that the GEV and the GP distributions are better suited to yield accurate flood quantiles in VRB. Hailegeorgis and Alfredsen [19] performed RFFA using LMO for annual maximum series of mean daily streamflow observations for reliable prediction of flood quantiles. Their studies revealed that both GEV and GP distributions were acceptable to fit for the homogeneous pooling groups of 20 catchments among the five distributions used in the study. Thus, the studies reported didn't suggest applying a particular distribution for FFA for different region or country. This apart, when different distributions are used for estimation of MFD, a common problem is encountered as regards the issue of best model fits for a given set of data. This can be answered by formal statistical procedures involving Goodness-of-Fit (GoF) and model performance analysis; and the results are quantifiable and reliable [20]. Qualitative assessment is made from the plot of the observed and estimated MFD. For quantitative assessment on MFD within in the observed range, Chi-square (χ^2) and Kolmogorov-Smirnov (KS) tests are applied. In addition to GoF tests, the performance of the probability distribution models used in frequency analysis of AMD is evaluated by Model Performance Indicators (MPIs) viz., Correlation Coefficient and Mean Absolute Percentage Error [21].

The paper attempts to present this work in comparison of five probability distributions (i.e., EXP, EV1, EV2, GEV and GP) is made to illustrate the applicability of GoF tests procedures and MPIs in identifying the best suitable distribution for estimation of MFD for river Ganga at Allahabad and Varanasi sites.

METHODOLOGY

The assessment of suitability of Probability Distribution Function (PDF) is essential in FFA. The procedures involved in carrying out FFA include (i) select PDFs for FFA (say, EXP, EV1, EV2, GEV and GP); (ii) select parameter estimation methods (say, MoM and LMO); (iii) select quantitative GoF test and MPIs; and (iv) conduct FFA and analyze the results obtained from the study.

Method of Moments

MoM is a technique for constructing estimators of the parameters based on matching the sample moments with the corresponding distribution moments [22-23]. The r^{th} central moment (μ_r) about the mean ($\bar{Q}$) of a continuous random variable Q is defined by:

$$\mu_r = E(Q-\bar{Q})^r = \int (Q-\bar{Q})^r f(Q)dQ \qquad (1)$$

where, f(Q) is PDF of a random variable Q. The second moment (μ_2) about $\bar{Q}$ is called as variance. Similarly, third and fourth moments (μ_3 and μ_4) about $\bar{Q}$ define the Coefficient of Skewness (C_S) and Kurtosis (C_K), which are as follows:

$$C_S=\mu_3/(\mu_2)^{3/2} \text{ and } C_K=[\mu_4/(\mu_2)^2]-3 \qquad (2)$$

L-Moments

LMOs are analogous to ordinary moments, which provide measures of location, dispersion, skewness, kurtosis and other aspects of the shape of probability distributions. But, LMOs are computed from linear combinations of the ordered data values [24]. LMO can be used as the basis of a unified approach to the statistical analysis adopting probability distributions. Moreover, the studies by Kyselý and Picek [25] revealed that the LMO is generally used in estimating the extreme values (i.e., discharge, rainfall, temperature, wind speed, etc) with the data series having less sample data and modified LMO for the data series having sufficiently large sample data. According to CWC [26], LMOs have the following advantages:

i) They characterize a wider range of probability distributions than conventional moments.
ii) They are less sensitive to outliers in the data.
iii) They approximate their asymptotic normal distribution more closely.
iv) They are nearly unbiased for all combinations of sample sizes and populations.

LMO thus would be useful in providing accurate quantile estimates of hydrological data in developing countries where small sample size typically exists. LMO is a linear combination of probability weighted moments. Let Q_1, Q_2, ...,Q_N be a conceptual random sample of size N and $Q_{1N} \le Q_{2N} \le ..... \le Q_{NN}$ denote the corresponding order statistics. The r+1th LMO defined by Hosking and Wallis [27] is:

$$l_{r+1} = \sum_{k=0}^{r} \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!} b_k \qquad (3)$$

where, l_{r+1} is the r+1th sample moment and b_k is an unbiased estimator with

$$b_k = N^{-1} \sum_{i=k+1}^{N} \frac{(i-1)(i-2).....(i-k)}{(N-1)(N-2).....(N-k)} Q_{iN} \qquad (4)$$

The first two sample LMOs are expressed by:

$$l_1 = b_0 \text{ and } l_2 = 2b_1 - b_0 \qquad (5)$$

The details of quantile functions and parameters of five probability distributions considered in the study are presented in Table 1.

In Table 1, F(Q) (or F) = P = 1-(1/T) is the cumulative distribution function (CDF) of Q; ξ, α, k are the location, scale and shape parameters respectively; ϕ^{-1} is the inverse of the standard normal distribution function and $\phi^{-1} = (P^{0.135} - (1-P)^{0.135})/0.1975$; μ (or $\bar{Q}$), σ (or S_Q) and C_S (or ψ) are the average, standard deviation and Coefficient of Skewness of the AMD; sign(k) is plus or minus 1 depending on the sign of k; Q_T is the estimated MFD by probability distribution corresponding to return period (T) [28].

Goodness-of-Fit Tests

GoF tests are essential for checking the adequacy of probability distributions to the AMD series in the estimation of MFD. Out of a number GoF tests available, the widely accepted GoF tests are χ^2 and KS, which are used in the study. The theoretical descriptions of which are given as below:

χ^2 test statistic:

$$\chi^2 = \sum_{j=1}^{NC} \frac{(O_j(Q) - E_j(Q))^2}{E_j(Q)} \qquad (6)$$

where, $O_j(Q)$ is the observed frequency value of j^{th} class, $E_j(Q)$ is the expected frequency value of j^{th} class and NC is the number of frequency classes. The rejection region of χ^2 statistic at the desired significance level (η) is given by $\chi^2_C \ge \chi^2_{1-\eta,NC-m-1}$. Here, m denotes the number of parameters of the distribution and χ^2_C is the computed value of statistic for the PDF.

KS test statistic:

$$KS = \underset{i=1}{\overset{N}{Max}}(F_e(Q_i) - F_D(Q_i)) \qquad (7)$$

where, $F_e(Q_i)$=i/(N+1) is the empirical CDF of Q_i, $F_D(Q_i)$ is the computed CDF of Q_i, Q_i is the observed MFD for i^{th} observation and N is the number of observations.

Test criteria: If the computed values of GoF tests statistic given by the distribution are less than that of the theoretical values at the desired level of significance then the distribution is considered to be acceptable for estimation of MFD at that level.

Table1. *Quantile functions and parameters of probability distributions*

S. No.	Distri-bution	Quantile function(Q_T)	Parameters by	
			MoM	LMO
1	EXP	$Q_T = \xi - \alpha \ln(1-F)$	$\xi = \overline{Q} - \alpha\,;\ \alpha = S_Q$	$\xi = l_1 - 2l_2\,;\ l_2 = \alpha/2$
2	EV1	$Q_T = \xi - \alpha \ln(-\ln(1-F))$	$\xi = \overline{Q} - 0.5772157\alpha \quad \alpha = \left(\sqrt{6}/\pi\right) S_Q$	$\xi = l_1 - 0.5772157\alpha$ $\alpha = l_2 / \log 2$
3	EV2	$Q_T = \alpha e^{(-\ln(-\ln(F))/k}$	By using the logarithmic transformation of the observed data, parameters of EV1 are initially obtained by MoM and LMO; and further used to determine the parameters of EV2 from $\alpha = e^{\xi}$ and k=1/(scale parameter of EV1).	
4	GEV	$Q_T = \xi + (\alpha(1-(-\ln F)^k)/k)$	$\overline{Q} = \xi + (\alpha(\Gamma(1+k)-1)/k)$ $S_Q = (\alpha/k)\left\{\Gamma(1+2k) - \Gamma(1+k)^2\right\}^{1/2}$ $\psi = (\operatorname{sign} k)\dfrac{\Gamma(1+3k) + 3\Gamma(1+k)(1+2k) - 2\Gamma^3(1+k)}{\left\{\Gamma(1+k) - \Gamma^2(1+k)^2\right\}^{1/2}}$	$z = (2/(3+t_3) - (\ln 2/\ln 3)$ $t_3 = (2(1-3^{-k})/(1-2^{-k})) - 3$ $k = 7.8590z + 2.9554z^2$ $\alpha = l_2 k/(1-2^{-k})\Gamma(1+k)$ $\xi = l_1 + (\alpha(\Gamma(1+k)-1)/k)$
5	GP	$Q_T = \xi + (\alpha(1-(1-F)^k)/k)$	$\overline{Q} = \xi + (\alpha/(1+k))$ $S_Q = \alpha^2/(1+2k)(1+k)^2$ $C_S = 2(1-k)(1+2k)^{1/2}/(1+3k)$	$\xi = l_1 + l_2(k+2)\,;\ t_3 = (1-k)/(3+k)$ $k = (4/(t_3+1)) - 3$ $\alpha = (1+k)(2+k)l_2$

Diagnostic Test

The selection of a suitable probability distribution for estimation of MFD is also carried out through model performance analysis using MPIs viz., Correlation Coefficient (CC) and Mean Absolute Percentage Error (MAPE). The theoretical expressions of CC and MAPE are given as below:

$$CC = \frac{\sum_{i=1}^{N}\left(Q_i - \overline{Q}\right)\left(Q_i^* - \overline{Q^*}\right)}{\sqrt{\left(\sum_{i=1}^{N}\left(Q_i - \overline{Q}\right)^2\right)\left(\sum_{i=1}^{N}\left(Q_i^* - \overline{Q^*}\right)^2\right)}} \quad (8)$$

$$MAPE = \left((1/N)\sum_{i=1}^{N}\left|\frac{Q_i - Q_i^*}{Q_i}\right|\right)*100 \quad (9)$$

where, Q_i^* is the predicted MFD for i^{th} observation by PDF, $\overline{Q}$ is the average value of observed MFD and $\overline{Q^*}$ is the average value of predicted MFD.

Selection criteria: The distribution with higher CC value (say, CC>0.9) and least MAPE is identified as better suited distribution in comparison with the other distributions for estimation of MFD.

Estimation of Peak Flood Discharge

A study was carried out to estimate the MFD adopting five probability distributions for fitting the AMD data from river Ganga at Allahabad and Varanasi gauging sites has been considered. The Allahabad gauging site is located between the latitude of 25° 23′ 35″ N and longitude of 81° 54′ 59″ E. The catchment area of the Allahabad gauging site is 463971 km^2. The average annual rainfall and temperature recorded at Allahabad site is 1027 mm and 25.7° C respectively. Likewise, the Varanasi gauging site is located between the latitude of 25° 19′ 25″ N and longitude of 83° 02′ 15″ E. The catchment area of the Varanasi gauging site is 489087 km^2. The average annual rainfall and temperature recorded at Varanasi site is about 1000 mm and 26.1° C respectively [29]. A map of Ganga river basin with the locations of Allahabad and Varanasi gauging sites [30] is presented in Figure 1.

In this paper, the daily stream flow data for the period 1986 to 2005 for Allahabad and Varanasi is used. The series of AMD is derived from the daily stream flow data and used in FFA. MoM and LMO have been used to determine the parameters of the probability distributions of five probability distributions (i.e., EXP, EV1, EV2, GEV and GP). The descriptive statistics of observed values of AMD for Allahabad and Varanasi is presented in Table 2. From Table 2, it is noted that the average AMD recorded at Varanasi is higher than that the corresponding value of Allahabad and the difference between the average AMD is computed as 1309.5 m^3/s.

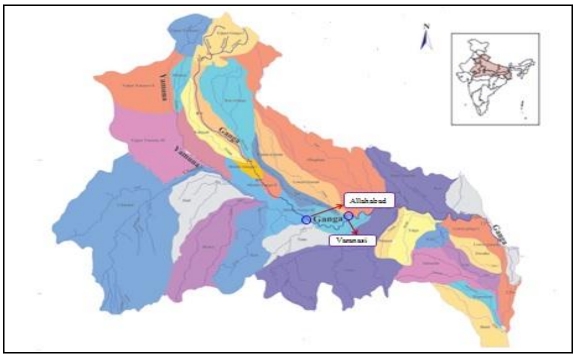

Figure1. *Location map of the study area*

DISCUSSION OF RESULTS

The procedures described above for estimating MFD were implemented adopting computer codes and used in FFA. The program computes the parameters of the five probability distributions with MoM and LMO methods which are employed for estimation of MFD for different return periods. The adequacy of fitting distributions was performed through GoF tests statistic and MPIs for the data under study.

Intercomparison of Estimates of MFD

The parameters of the adopted five probability distributions in this study were determined by MoM and LMO; and used for estimation of MFD for river Ganga at Allahabad and Varanasi. The results of MFD estimates are presented in Tables 3 and 4.

The parameters of the distributions were also used to determine the predicted value of maximum discharge at consecutive years based on the probabilities of observed value of maximum discharge, and used for generating the time series plots (Figure 2 to 5).

Table2. *Descriptive statistics of observed values of AMD for Allahabad and Varanasi*

Gauging site	Statistical parameters				
	Average (m^3/s)	SD (m^3/s)	CV (%)	C_S	C_K
Allahabad	26644.2	8532.5	32.0	-0.004	-1.236
Varanasi	27953.7	7776.5	27.8	-0.115	-0.143
SD: Standard Deviation; CV: Coefficient of Variation [CV (%) = (SD/Average) x 100]					

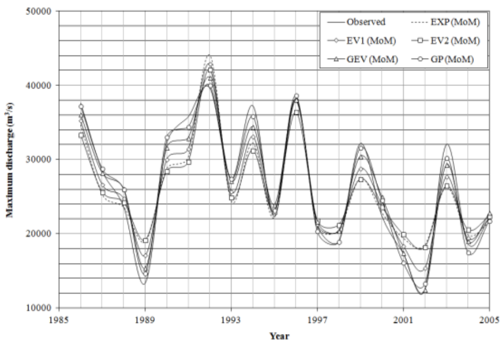

Figure2. *Plots of observed and predicted values of maximum discharge by probability distributions (using MoM) for Allahabad*

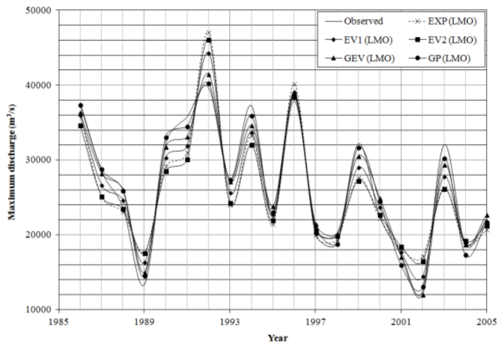

Figure3. *Plots of observed and predicted values of maximum discharge by probability distributions (using LMO) for Allahabad*

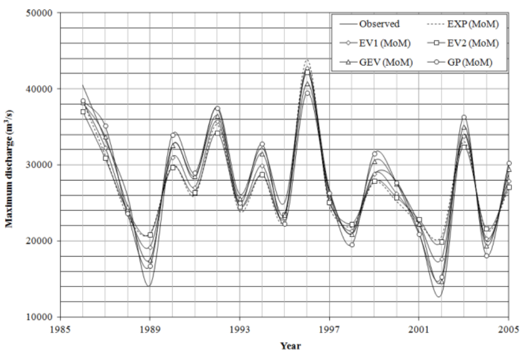

Figure4. *Plots of observed and predicted values of maximum discharge by probability distributions (using MoM) for Varanasi*

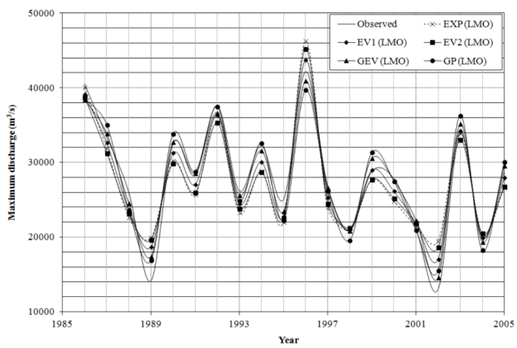

Figure5. *Plots of observed and predicted values of maximum discharge by probability distributions (using LMO) for Varanasi*

Table3. *MFD estimates computed by probability distributions for Allahabad*

Return period (year)	Estimated MFD (m^3/s)									
	MoM					LMO				
	EXP	EV1	EV2	GEV	GP	EXP	EV1	EV2	GEV	GP
2	24026	25242	24582	26585	26656	23575	25124	23861	26585	26635
5	31844	32785	30942	34069	35514	32739	33301	31611	34310	35643
10	37758	37780	36034	37871	38456	39671	38715	38081	38234	38654
20	43673	42570	41704	40842	39923	46604	43908	45528	41298	40162
50	51491	48771	50389	43900	40801	55767	50630	57371	44451	41070
100	57405	53418	58063	45724	41092	62700	55667	68224	46331	41373
200	63319	58048	66870	47220	41238	69632	60685	81079	47873	41525
500	71138	64156	80567	48798	41324	78795	67306	101816	49499	41616
1000	77052	68773	92751	49750	41353	85728	72310	120939	50479	41647

Table4. *MFD estimates computed by probability distributions for Varanasi*

Return period (year)	Estimated MFD (m^3/s)									
	MoM					LMO				
	EXP	EV1	EV2	GEV	GP	EXP	EV1	EV2	GEV	GP
2	25568	26676	26114	28065	28314	25206	26593	25610	28065	28159
5	32693	33551	32080	34789	36087	33410	33913	32652	34884	36051
10	38083	38103	36762	38088	38391	39617	38760	38349	38230	38513
20	43474	42469	41894	40597	39435	45823	43409	44746	40774	39677
50	50599	48121	49615	43103	39997	54027	49427	54635	43317	40334
100	55989	52356	56320	44554	40164	60233	53937	63454	44789	40539
200	61380	56575	63901	45714	40239	66439	58430	73656	45966	40636
500	68505	62142	75486	46902	40280	74644	64358	89667	47172	40691
1000	73896	66350	85617	47597	40292	80850	68838	104038	47878	40708

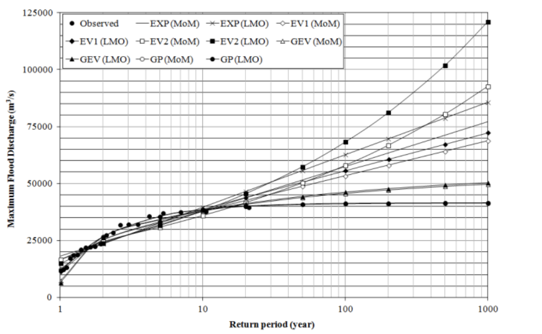

Figure6. *Plots of observed and estimated MFD by probability distributions (using MoM and LMO) for Allahabad*

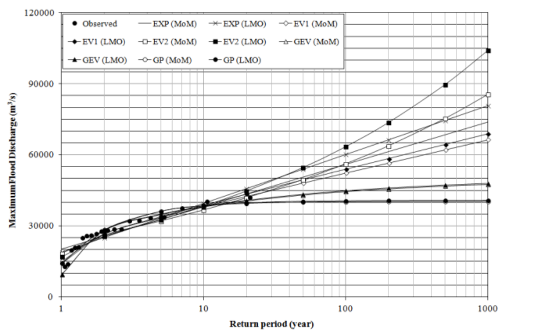

Figure7. *Plots of observed and estimated MFD by probability distributions (using MoM and LMO) for Varanasi*

From Tables 3 and 4, it is noticed that the estimated MFD by EV2 (using LMO) was higher when compared to the corresponding values of other distributions (using MOM and LMO) for the return period of 50-year and above. The probability plots of the results of the five distributions adopted are presented in Figures 6 and 7 respectively. The descriptive statistics of predicted values of AMD by probability distributions for Allahabad and Varanasi are presented in Tables 5 and 6.

Table5. *Descriptive statistics of predicted values of AMD for Allahabad*

Statistical parameters	MoM					LMO				
	EXP	EV1	EV2	GEV	GP	EXP	EV1	EV2	GEV	GP
Average (m^3/s)	26027.5	26286.9	26000.1	26653.4	26645.4	25921.4	26256.9	25731.2	26654.2	26642.9
SD (m^3/s)	6934.4	7257.3	6151.7	7734.1	8327.5	8127.9	7866.7	7556.1	7984.8	8451.8
CV (%)	26.6	27.6	23.7	29.0	31.2	31.3	30.0	29.4	30.0	31.7
Skewness	1.210	0.638	1.127	0.027	-0.004	1.210	0.638	1.240	0.026	0.003
Kurtosis	1.057	-0.023	1.131	-0.641	-1.201	1.057	-0.023	1.473	-0.641	-1.199
SD: Standard Deviation; CV: Coefficient of Variation										

Table6. *Descriptive statistics of predicted values of AMD for Varanasi*

Statistical parameters	MoM					LMO				
	EXP	EV1	EV2	GEV	GP	EXP	EV1	EV2	GEV	GP
Average (m^3/s)	27391.7	27628.1	27392.1	28000.2	27993.6	27306.6	27607.1	27213.0	28000.6	27976.8
SD (m^3/s)	6320.1	6614.3	5753.4	7058.8	7605.7	7276.7	7042.9	6822.8	7157.0	7580.6
CV (%)	23.1	23.9	21.0	25.2	27.2	26.6	25.5	25.1	25.6	27.1
Skewness	1.210	0.638	1.074	-0.047	-0.115	1.210	0.638	1.155	-0.047	-0.066
Kurtosis	1.057	-0.023	0.979	-0.651	-1.215	1.057	-0.023	1.214	-0.651	-1.213

From the values of descriptive statistics, as given in Tables 2, 5 and 6, the following observations were made:

i) For Allahabad, the percentage of variation on the predicted average value of maximum discharge by five probability distributions with reference to the observed average value of maximum discharge is found to be in the range of 0.0% to 2.4% when MoM is applied for determination of parameters of probability distributions; and which it is in the range of 0.0% to 3.4% when LMO is used.

ii) For Varanasi, the percentage of variation on the predicted average value of maximum discharge by probability distributions with reference to the observed average value of maximum discharge is found to be in the range of 0.1% to 2.0% for MoM whereas 0.1% to 2.7% for LMO.

iii) For Allahabad, it is noticed that the average value of predicted maximum discharge by GEV distribution (using MoM and LMO) is closer to the average value of observed maximum discharge.

iv) For Varanasi, it is noted that the average value of predicted maximum discharge by GP distribution (using MoM and LMO) is nearer to the average value of observed maximum discharge.

Analysis of Results Based on GoF Tests

The adequacy of fitting different PDFs adopted in frequency analysis of AMD data was performed by adopting GoF tests; and the tests results are presented in Table 7. In the present study, the degree of freedom was considered as one for 3-parameter distributions and two for 2-parameter distributions while computing the χ^2 statistic values.

Table7. *Theoretical and Computed values of GoF tests statistic by probability distributions*

Distri-bution	Theoretical value GoF tests statistic at 5% level		Computed values of GoF tests statistic							
			Allahabad				Varanasi			
			MoM		LMO		MoM		LMO	
	χ^2	KS	χ^2	KS	χ^2	KS	χ^2	KS	χ^2	KS
EXP	5.99	0.294	1.500	0.185	1.500	0.166	5.500	0.192	4.500	0.208
EV1	5.99	0.294	3.000	0.159	3.000	0.147	5.500	0.126	5.500	0.133
EV2	5.99	0.294	7.500	0.209	6.500	0.190	5.500	0.156	5.500	0.189
GEV	3.84	0.294	2.000	0.110	2.000	0.105	2.500	0.084	2.500	0.084
GP	3.84	0.294	4.000	0.079	4.000	0.077	3.500	0.102	3.500	0.100

From GoF test results, as given in Table 7, the following observations were made:

i) Results of χ^2 test didn't infer the adequacy of fitting the EV2 and GP distributions (using MoM and LMO) for frequency analysis of AMD for Allahabad.

ii) Results of χ^2 test statistic indicated the applicability of EXP, EV1, EV2, GEV and GP distributions (using MoM and LMO) for frequency analysis of AMD for Varanasi.

iii) Results of KS test indicated the use all five probability distributions (using MoM and LMO) adopted in frequency analysis of AMD for Allahabad and Varanasi.

Analysis of Results Based on MPIs

For the selection of the best suitable distribution for estimation of MFD, the values of MPIs were computed by the probability distributions (using MoM and LMO) through Eqs. (8) and (9); and the results are presented in Table 8.

Table8. *CC and MAPE values computed by probability distributions (using MoM and LMO)*

Distri-bution	Allahabad				Varanasi			
	MoM		LMO		MoM		LMO	
	CC	MAPE (%)	CC	MAPE (%)	CC	MAPE (%)	CC	MAPE (%)
EXP	0.917	11.8	0.917	10.7	0.927	11.5	0.927	10.4
EV1	0.962	8.3	0.962	7.1	0.972	7.4	0.972	6.8
EV2	0.926	12.9	0.916	10.6	0.947	11.1	0.941	10.1
GEV	0.984	5.0	0.984	4.8	0.988	4.6	0.988	4.4
GP	0.991	4.1	0.991	4.1	0.972	6.3	0.973	6.3

From the results of MPIs, as given in Table 8, a critical analysis of was conducted and the following observations were made:

i) The MAPE values of 4.1% (using GP) for Allahabad and 4.4% (using GEV) for Varanasi are lowest when MoM and LMO are applied for determination of parameters of the distributions.

ii) χ^2 test results indicate that the EV2 and GP distributions (using MoM and LMO) are not adequate for estimation of MFD at Allahabad.

iii) By eliminating the MAPE values of these two distributions from the selection of probability distribution for estimation of MFD, it can be seen that the MAPE of GEV (using LMO) is noted as the second minimum next to GP (using MoM and LMO) for Allahabad.

iv) The CC between the observed and predicted values of maximum discharge by probability distributions (using MoM and LMO) is observed to vary from 0.917 to 0.991 for Allahabad and 0.927 to 0.988 for Varanasi thus indicating adequacy.

v) The CC values indicated that there is generally good correlation between the observed and predicted values of maximum discharge by probability distributions adopted in the study.

The above analysis of results obtained from quantitative assessment using GoF tests and MPIs offered diverging inferences; and thus called for qualitative assessment using time series plots (Figures 2 to 5) and probability plots of the results (Figures 6 and 7). This analysis shows the GEV (using LMO) distribution as better suited amongst five probability distributions adopted for estimation of MFD at Allahabad and Varanasi gauging sites.

CONCLUSIONS

The paper presents briefly the study carried out for estimation of MFD by adopting FFA using a computer aided procedure for determination of parameters of five probability distributions (using MoM and LMO) for river flow data at Allahabad and Varanasi. The intercomparison of results was performed and the following conclusions were drawn from the study:

i) For the return period of 10-year and above, it was found that the estimated MFD by EXP, EV1, EV2, GEV and GP distributions (using LMO) are higher than the corresponding values of MoM of these distributions for both Allahabad and Varanasi.

ii) From the χ^2 test results, it could be seen that the EXP, EV1 and GEV distributions (using MoM and LMO) are acceptable for frequency analysis of AMD data for Allahabad.

iii) The χ^2 test results confirmed the applicability of EXP, EV1, EV2, GEV and GP distributions (using MoM and LMO) for frequency analysis of AMD data for Varanasi.

iv) The KS test results indicated that all five probability distributions (using MoM and LMO) adopted in FFA is acceptable for estimation of MFD at Allahabad and Varanasi gauging sites.

v) The CC values obtained from probability distributions (using MoM and LMO) vary from 0.917 to 0.991 for Allahabad and 0.927 to 0.988 for Varanasi.

vi) On the basis of results obtained quantitative assessment, the study suggested that the GEV (using LMO) distribution as better suited amongst five distributions studied for estimation of MFD at Allahabad and Varanasi.

The study recommends the MFD values for different return periods computed by GEV (using LMO) distribution for planning and design of water resources structures and flood protection works on the river in the region of Allahabad and Varanasi gauging sites. The qualitative assessment aided in crystallizing the outcomes for the selection of a distribution for MFD, say GEV (using LMO).

ACKNOWLEDGEMENTS

The authors are grateful to Dr. (Mrs.) V.V. Bhosekar, Director, Central Water and Power Research Station, Pune, for encouragement during conduct of the studies and also accordingly permission to publish this paper. The authors are thankful to Central Water Commission, New Delhi, for making available the stream flow data required for the study.

REFERENCES

[1] Gubareva, T.S., and Gartsman, B.I. (2010). Estimating distribution parameters of extreme hydrometeorological characteristics by L-Moment method. Water Resources, 37, 437–445.

[2] Naghavi, B., Yu, F.X., and Singh, V.P. (1993). Comparative evaluation of frequency distributions for Louisiana extreme rainfall. Water Resources Bulletin, 29, 211–219.

[3] CEH. (2001). FLOODS version 1.1, Regional flood frequency analysis software manual. Water Resources Section, Centre for Ecology and Hydrology (CEH), Wallingford, U.K.

[4] Hosking, J.R.M. (1990). L-moments: Analysis and estimation of distributions using linear combinations of order statistics. Royal Statistical Society (Series B), 52, 105-124.

[5] Kjeldsen, T.R., Smithers, J.C., and Schulze, R.E. (2002). Regional flood frequency analysis in the KwaZulu-Natal province, South Africa, using the index-flood method. Journal of Hydrology, 255, 194–211.

[6] Kumar, R., Chatterjee, C., Kumar, S., Lohani, A.K., and Singh, R.D. (2003). Development of regional flood frequency relationships using L-moments for Middle Ganga Plains Subzone 1(f) of India. Water Resources Management, 17, 243–257.

[7] Yue, S., and Wang, C.Y. (2004). Possible regional probability distribution type of Canadian annual stream flow by L-moments. Water Resources Management, 18, 425–438.

[8] Kumar, R., and Chatterjee, C. (2005). Regional flood frequency analysis using L-Moments for north Brahmaputra region of India. Hydrologic Engineering, 10, 1–7.

[9] Atiem, I.A., and Harmancioglu, N.B. (2006). Assessment of regional floods using L-moments approach: the case of the River Nile. Water Resources Management, 20, 723–747.

[10] Abida, H., and Ellouze, M. (2008). Probability distribution of flood flows in Tunisia. Hydrology and Earth System Sciences, 12, 703-714.

[11] Saf, B. (2009). Regional flood frequency analysis using L-Moments for the West Mediterranean Region of Turkey. Water Resources Management, 23, 531-551.

[12] Bhuyan, A., Borah, M., and Kumar, R. (2010). Regional flood frequency analysis of North-Bank of the River Brahmaputra by using LH-Moments. Water Resources Management, 24, 1779-1790.

[13] Malekinezhad, H., Nachtnebel, H.P., and Klik, A. (2011). Regionalization approach for extreme flood analysis using L-moments. Agricultural Science and Technology (Iran), 13 (Supplementary Issue), 1183–1196.

[14] Badreldin, G.H.H., and Feng, P. (2012). Regional rainfall frequency analysis for the Luanhe Basin using L-moments and cluster techniques. International Conference on Environmental Science and Development, 5-7 Jan 2012, Hong Kong,Vol. 1, pp. 126–135.

[15] Hailegeorgis, T., Thorolfsson, S.T., and Alfredsen, K. (2013). Regional frequency analysis of extreme precipitation with consideration of uncertainties to update IDF curves for the city of Trondheim. Journal of Hydrology, 498, 305–318.

[16] Haberlandt, U., and Radtke, I. (2014). Hydrological model calibration for derived flood frequency analysis using stochastic rainfall and probability distributions of peak flows. Hydrology and Earth System Sciences, 18, 353-365.

[17] Markiewicz, I., Strupczewski, W.G., Bogdanowicz, E., and Kochanek, K. (2015). Generalized exponential distribution in flood frequency analysis for Polish Rivers, PLoS ONE, 10, 1-15.

[18] Kossi, K., Barnabas, A.A., Bernd, D., and Fabien, C.C.H. (2016). Regional flood frequency analysis in the Volta river basin, West Africa. Hydrology Journal, 3, 1-15.

[19] Hailegeorgis, T.T., and Alfredsen, K. (2017). Department Regional flood frequency analysis and prediction in ungaugedbasins including estimation of major uncertainties for mid-Norway. Journal of Hydrology: Regional Studies, 9, 104-126.

[20] Zhang, J. (2002). Powerful goodness-of-fit tests based on the likelihood ratio. Journal of Royal Statistical Society, 64. 281-294.

[21] Chow, S.K.H., Lee, E.W.M., and Li, D.H.W. (2012). Short-term prediction of photovoltaic

energy generation by intelligent approach. Energy Building, 55(December issue), 660-667.

[22] Haktanir, T., and Horlacher, H.B. (1993). Evaluation of various distributions for flood frequency analysis. Hydrological Sciences Journal, 38, 15-32.

[23] Ghorbani, A.M., Ruskeep, A.H., Singh, V.P., and Sivakumar, B. (2010). Flood Frequency analysis using mathematica. Turkish Journal of Engineering and Environmental Sciences, 34, 171-188.

[24] Vogel, R.M., and Wilson, I. (1996). The probability distribution of annual maximum, minimum and average stream flow in the United States. Journal of Hydrologic Engineering, 1, 69-76.

[25] Kyselý, J. and Picek, J. (2007). Regional growth curves and improved design value estimates of extreme precipitation events in the Czech Republic. Climate Research, 33, 243-255.

[26] CWC. (2010). Development of hydrological design aids (surface water) under HP-II: State of the art report. Consulting Engineering Services (India) in Association with HR Wallingford, New Delhi.

[27] Hosking, J.R.M., and Wallis, J.R. (1993). Some statistics useful in regional frequency analysis. Water Resources Research, 29, 271-281.

[28] Rao, A.R., and Hameed, K.H. (2000). Flood frequency analysis. CRC Press, Florida, USA.

[29] CWC. (2012). Hydrological observation stations in India under Central Water Commission (CWC), New Delhi.

[30] NRCD. (2009). Status paper on river Ganga: Status on environment and water quality. National River Conservation Directorate (NRCD), Ministry of Environment and Forests, August 2009.

Citation: *N. Vivekanandan and C. Ramesh, " Intercomparison of Estimates of Maximum Flood Discharge Using Quantitative and Qualitative Assessments Adopting L-Moments in Parameter Estimation of Probability Distributions", International Journal of Emerging Engineering Research and Technology, vol. 5, no. 9, pp. 37-46, 2017.*