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Damping of Vibrations of Dissipative-Inhomogeneous Multi-Layer Plates and Shells Interacting with the Medium

I.I. Safarov[1], M.Kh. Teshaev[2]

[1]*Tashkent Institute of Chemistry and Technology, Tashkent, Republic of Uzbekistan.*
[2]*Bukhara Technological- Institute of Engineering, Bukhara, Republic of Uzbekistan*

***Corresponding Author:** I.I. Safarov, Tashkent Institute of Chemistry and Technology, Tashkent, Republic of Uzbekistan*

ABSTRACT

The destabilizing effect of vibration significantly increases during the resonance oscillations of the elements of engineering constructions. The amplitudes of displacement, velocity, and acceleration are greatly increased and can exceed tens of times the amplitudes of the perturbing oscillations. This article presents calculations of dynamic behavior (semi-analytical method) and stress-strain state of laminated lamellar and tubular structures.

Keywords: *base, three-layer, plates, shells, oscillations, Winkler, Pasternak*

INTRODUCTION

Layered structures are used as elements of the hulls of aircraft and space vehicles, building panels, electronic boards, engineering elements, and other vibrations of three-layer bodies in a dissipative homogeneous case, was considered in [1,2,3,4].

In the framework of the dynamic theory of viscoelasticity, oscillations of a three-layer plate (or shells) interacting with elastic (viscoelastic) Winkler and Pasternak bases were investigated [5,6,7].

Mathematical formulation of the problem of oscillations of dissipative-inhomogeneous layered plates and shells interacting with the medium.

We consider oscillations of dissipative-inhomogeneous three-layer (or two-layer) plates and shells interacting with the medium (Fig. 1). In thin isotropic viscoelastic bearing layers of thickness h_k the Kirchhoff-Love hypotheses are applied in a relatively thick aggregate ($h_p = 2a$). Exact relationships of the theory of elasticity are valid for it. On the boundaries between layers, the continuity conditions of displacements are used. Deformations are small. For the requested functions are accepted u_α^k, v_α^k - tangential displacements and deflections of the points of the middle surface of the bearing layers. The equations of motion of a mechanical system, taking into account the boundary conditions, are obtained on the basis of the principle of possible displacements:

$$\delta A_F + \delta A_I + \delta A_W = 0, \qquad (1)$$

where δA_F - variation of external forces:

$$\delta A = \int_0^{l_1}\int_0^{l_2}\sum_{k=1}^{2}\left[\begin{gathered}\left((q_\alpha^k + q_{\alpha r}^k)\left(\delta u_\alpha^k + \frac{(-1)^{k+1}h_k}{2}\delta\psi_\alpha^k\right) + (q_3^k + q_{3r}^k)\delta W^k\right)\times \\ \times(1+(-1)^{k+1}k_1(c+h_k)(1+(-1)^{k+1}k_2(c+h_k))\end{gathered}\right]$$
$$H_1H_2dx_1dx_2$$

δA_W - variations in the work of internal viscoelastic forces:

$$\delta W = \int_0^{l_1}\int_0^{l_2}\left[\begin{gathered}\sum_{k=1}^{3}\int_{h_k}\sigma_{\alpha\beta}^k\delta\varepsilon_{\alpha\beta}^{kz}(1+k_1z)(1+k_2z)\,dz + \\ + \int_{h_3}(2\sigma_{\alpha 3}^3\delta\varepsilon_{\alpha 3}^{3z} + \sigma_{33}^3\delta\varepsilon_{33}^{3z})(1+k_1z)(1+k_2z)dz\end{gathered}\right]H_1H_2dx_1dx_2$$

δA_I - variation of the inertia forces:

$$\delta A_I = \sum_{k=1}^{3}\int_0^{l_1}\int_0^{l_2}\int_{h^k}[\rho_k(\ddot{w}^k\delta w^k + \ddot{u}_\alpha^k\delta u_\alpha^k)](1+k_1z)(1+k_2z)$$
$$H_1H_2dzdx_1dx_2$$

Here l_α - linear dimension of a plate or shell in the direction of the axis $x_\alpha (\alpha = 1,2)$; H_α, k_α - The Lame coefficients and the principal curvature of the middle surface for shells, q_α^k q_3^k - distribution of external loads applied to the outer surfaces of the carrier layers; ρ_k - density.

On the boundaries with rigid attachment of the contact, conditions:

$$u_1^k = u_2^k = w^k = w_{1'1}^k = 0, \quad (x = 0, L; \quad k = 1,2)$$

After the corresponding transformations from the variation equation (1), we obtain six equations of motion for a three-layer viscoelastic plate and a shell of revolution associated with an elastic medium ($k = 1,2; i = 1,2,3$). Stresses and strains in the layers of a three-layer lamellar (or cylindrical layer) are connected by Hooke's law:

$$s_{\alpha\beta}^k = 2\tilde{G}^k \vartheta_{\alpha\beta}^k = 2G^k\left[\vartheta_{\alpha\beta}^k - \int_{-\infty}^{t} R'^k_{\alpha\beta}(t-\tau)\vartheta_{\alpha\beta}^k(\tau)d\tau\right],$$

$$\sigma^k = 3\tilde{K}_k \varepsilon^k = 3K_k\left[\varepsilon^k - \int_{-\infty}^{t} R'^k_{\varepsilon}(t-\tau)\varepsilon^k(\tau)d\tau\right], \qquad (2)$$

where $s_{\alpha\beta}^k$ - deviatorial, σ^k, ε^k - Ball parts of stress and strain tensors in the k-th layer, G^k, K_k - instantaneous shear and volume strain modules, $R'^k_{\alpha\beta}(t-\tau), R'^k_{\varepsilon}(t-\tau)$ -respectively, the relaxation nucleus. The distributed load is applied to the outer surfaces of the carrier layers q_1^k and the reaction of the medium [8]

$$q_{3r}^k = -\tilde{k}_0^k w^k = -k_0^k\left[w^k - \int_{-\infty}^{t} R_k(t-\tau)w^k(\tau)d\tau\right], \quad q_{\alpha r}^k = 0$$

where w^k - deflection, k_0^k - coefficient of instantaneous stiffness of the environment, $R_k(t-\tau)$ - relaxation core.

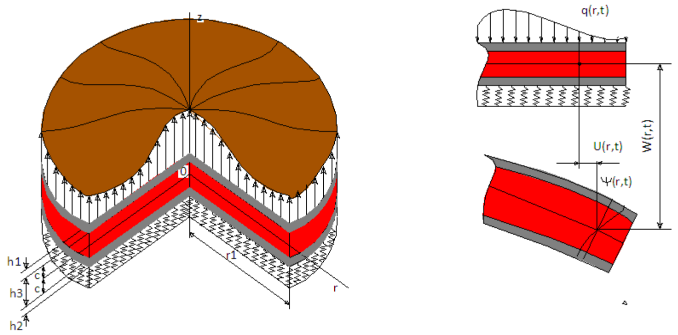

Fig1. *Calculation scheme of a three-layer plate on an elastic base*

The resolving equations in the displacements for the system under consideration follow from the equations of motion (1), after expressing the internal forces through (2) the quantities u_α^m, w^m and accounting for the reaction of the base

$$L_\alpha^m(u_\alpha^m, w^m) - b_\alpha^m \ddot{u}_\alpha^m = -L_{\alpha q}^m,$$
$$L_3^m(u_\alpha^m, w^m) - b_3^m \ddot{w}^m = -L_{3q}^m \quad (m, \alpha = 1,2) \qquad (3)$$

$$L_\alpha^m = \sum_{k=1}^{2}[(a_{m\alpha1}^k \frac{\partial^2}{\partial x_\alpha^2} + a_{m\alpha2}^k \frac{\partial^2}{\partial x_\beta^2} + a_{m\alpha3}^k)u_\alpha^k + a_{m\alpha4}^k \frac{\partial^2 u_\alpha^k}{\partial x_\alpha \partial x_\beta} +$$
$$(a_{m\alpha5}^k \frac{\partial}{\partial x_\alpha} + a_{m\alpha6}^k \frac{\partial^3}{\partial x_\alpha^3} + a_{m\alpha7}^k \frac{\partial^3}{\partial x_\alpha \partial x_\beta^2})w^k],$$

$$L_3^m = \sum_{\alpha,k=1}^{2}[(a_{m31}^{\alpha k} \frac{\partial^4}{\partial x_\alpha^4} + a_{m32}^k \frac{\partial^4}{\partial x_1^2 \partial x_2^2} + a_{m33}^{\alpha k} \frac{\partial^2}{\partial x_\alpha^2} + a_{m34}^k - Rm_m k_0^m \delta_{mk})w_k +$$
$$+ (a_{m35}^{\alpha k} \frac{\partial^3}{\partial x_\alpha^3} + a_{m36}^{\alpha k} \frac{\partial}{\partial x_\alpha} + a_{m37}^{\alpha k} \frac{\partial^3}{\partial x_\alpha \partial x_\beta^2})u_\alpha^k], \quad (m, \alpha, \beta = 1,2; \quad \alpha \neq \beta)$$

where

$$L_{1q}^m = m_m R q_1^m, \quad L_{2q}^m = (R + 0{,}5h_m c_2^m) m_m q_2^m, \quad m_m = 1 \pm (c + h_m) R^{-1},$$ A

$$L_{3q}^m = Rm_m[q_3^m \pm 0{,}5h_m(q_{1'1} + R^{-1}c_2^m q_{2'2}^m)].$$

system of partial differential equations describing the forced transverse vibrations of a circular three-layer plate connected with an elastic inertial-free base, without taking into account the compression and inertia of the normal rotation in the layers, is derived from the variation principle, taking into account the variation of inertia forces, in the axisymmetric case has the form

$$\begin{cases} L_2(a_1 u + a_2\psi - a_3 w,_r) = 0 \\ L_2(a_2 u + a_4\psi - a_5 w,_r) = 0 \\ L_3(a_3 u + a_5\psi - a_6 w,_r) = M_0\ddot{w} - k_0 w = -q \end{cases}$$

where

$M_0 = \rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3$; L_2, L_3 – differential operators;

a_i - coefficients,

$$a_1 = \sum_{k=1}^{3} h_k K_k^+; \quad a_2 = c(h_1 K_1^+ - h_2 K_2^+); \quad K_k^+ \equiv K_k + \frac{4}{3}G_k;$$

$$a_3 = h_1(c + \frac{1}{2}h_1)K_1^+ - h_2(c + \frac{1}{2}h_2)K_2^+); \quad a_4 = c^2(h_1 K_1^+ + h_2 K_2^+ + \frac{2}{3}cK_3^+);$$

$$a_5 = c\left[h_1(c + \frac{1}{2}h_1)K_1^+ + h_2(c + \frac{1}{2}h_2)K_2^+) + \frac{2}{3}c^2 K_3^+\right];$$

$$a_6 = h_1(c^2 + ch_1 + \frac{1}{3}h_1^2)K_1^+ + h_2(c^2 + ch_2 + \frac{1}{3}h_2^2)K_2^+ + \frac{2}{3}c^3 K_3^+;$$

$$L_2(g) \equiv \left(\frac{1}{r}(rg),_r\right),_r \equiv g,_{rr} + \frac{g,_r}{r} - \frac{g}{r^2}$$

$$L_3(g) \equiv \frac{1}{r}(rL_2(g)),_r \equiv g,_{rrr} + \frac{2g,_{rr}}{r} - \frac{g,_r}{r^2} + \frac{g}{r^3};$$

$G_k, K_k -$ modules of shear and volume deformation of the material of the k-th layer. The task of defining functions $u(r,t),\ \psi(r,t),\ w(r,t)$, is closed by adjoining (1) boundary and initial conditions:

$w(r,0) \equiv f(r);\quad w(r,0) \equiv g(r);$ Here δ_{mk} - Kronecker symbols, coefficients a_{ijk}^{lm} depend on the rigidity of the shell and plate, L_{iq}^{m} - external load parameter, (m=1,2),

$$u_\alpha^k = \sum_{m,n} \psi_{\alpha mn}^k(x,\varphi) T_{\alpha mn}^k(t)\,,\quad w_k = \sum_{m,n} \psi_{3mn}^k(x,\varphi) T_{3mn}^k(t)\,,$$

$$q_l^k = \sum_{m,n} \psi_{qlmn}^k(x,\varphi)\, q_{lmn}^k(t)\,,\quad (l = 1,2,3);\quad \alpha, k = 1,2$$

The system of integro-differential equations of motion in partial derivatives (3) is linear. To obtain the solution, we use the Bubnov-Galerkin method. Then we obtain the integro-differential equation:

$$[M]\{\ddot{T}\} + [PK](\{T\} - \int_0^t [R(t-\tau)]\{T(\tau)\} d\tau) = \{Q(t)\}, \qquad (4)$$

where $[M],[PK],[R(t-\tau)]$ - quadratic positive definite matrices, $\{T\}$, $\{Q(t)\}$ vector of a column of unknown displacements and external loads. When solving the system of integro-differential equations (4), the freezing method is applied, as was done in the first and second chapters. Then, instead of a system of integro-differential equations, we obtain a system of second-order differential equations with complex coefficients:

$$[M]\{\ddot{T}\} + [P\bar{K}]\{T\} = \{Q(t)\} \qquad (5)$$

We consider the intrinsic and forced vibrations of the above constructions. An algorithm and program for studying a mechanical system with a finite number of degrees of freedom and a system with distributed parameters on the basis of the Muller, Gauss methods, the method of the orthogonal Godunov run, the method, and the averaging method [9].

Algorithm for determining the frequency equation for multilayer plate and cylindrical bodies

When investigating free oscillations, the solution is sought in the form:

$$u_\alpha^k = \sum \Psi_{\alpha mn}^k(\vec{r}) e^{i\omega t}\,,\qquad l = 1,2,3;\ \alpha, \kappa = 1,2\,.$$

$$w^k = \sum \Psi_{3mn}^k(\vec{r}) e^{i\omega t}\,,$$

$$q_l^k = \sum \Psi_{qlmn}^k(\vec{r}) e^{i\omega t}\,,$$

(6)

where $\Psi_{\alpha mn}^k, \Psi_{3mn}^k, \Psi_{qlmn}^k$ и , ω - complex amplitude and complex frequency of oscillations. Substituting expressions (6) into system (5), we arrive at a complex generalized eigenvalue problem:

$$([PK] - \omega^2 [M])\{A\} = 0 \qquad (7)$$

When studying the processes of damped oscillations in elastic layer-homogeneous media with plane-parallel interfaces, it is first necessary to determine the frequency characteristics of these oscillations. Frequency characteristics are complex natural frequencies $\omega = \omega_R + i\omega_I$ and the corresponding proper forms. Here ω_I describes the damping of the oscillations in time. It is known that the quantities ω are related to the value of the root of the frequency equation

$\Delta(\omega, \xi)=0,$

where ω- complex frequency; ξ - physical - mechanical or geometric parameters of structures. Thus, in order to be able to calculate the frequency characteristics, it is necessary to carry out a qualitative investigation of the roots of equation (7) at points of the complex plane, and also to develop a method for their numerical determination. A qualitative investigation of the roots of the frequency equation was carried out in [10].

When solving problems of the frequency equation with complex input coefficients and roots, numerical methods are applied. Solving frequency equations with complex roots, which are often complex transcendental equations, is difficult even in the case of computers. Moreover, it is necessary to take into account that the problem must be repeated at different constant frequency values to obtain the phase velocity, or the wavelength. Three methods can be used to solve the frequency transcendental equations:

A) solution of the system of two transcendental equations;

B) the direct definition of complex roots by the quadratic interpolation method, or by another similar iterative method (for example, Muller's methods);

c) the approximate solution of the frequency equation by the small parameter method. The frequency equation with complex roots in special cases is reduced to the system of two transcendental equations by separating the real and imaginary parts. Of course, finding a pair of characteristic values, at which both equations are simultaneously satisfied, the process is difficult, but sometimes leads to a goal. In [11], the idea of a small parameter method was used to solve the frequency equation of stress waves propagating in a viscoelastic layer on an elastic half-space, the material of the layer corresponding to a Voigt-type rheological model. The attenuation and viscosity parameters are taken here as small parameters. But here we obtain a frequency equation for the elastic case, by means of which it is impossible to obtain dispersion regularities in a viscoelastic system.

More expedient is the direct definition of complex roots by Muller's methods. For complex roots, Muller's method simplifies calculations and provides faster convergence than the Barstow method if the roots are close to each other [11]. The Muller method uses quadratic interpolation, which leads to iteration of the form:

$$Z^{[j+1]} = Z^{[j]} - (Z^{[j]} - Z^{[j-1]})\frac{2C_j}{B_{jj}^2 4A_jC_j} signB,$$

where $A_j = g_i f_j - g_i (1+g_i)^2 f_{j-1} + g_i f_{j-2}$;

$B_j = (2g_i+1) f_j^2 - (1+g_i)^2 f_{j-1} + g_i f_{j-2}$;

$C_j = (g_i+1) f_j$; $f_j = f(z^{([j]})$;

$g_j = (z^{[j}-z^{[j-1]})/(z^{[j-1]}-z^{[j-2]})$; $j=0,1,2$.

To start the solution, we can put $z^{[0]} = z\ 00$; $z^{[1]} = z\ 01$; $z^{[2]} = z\ 02$; z_{00}, z_{01}, z_{02} – solution of elastic problems. Based on the latest modification, an algorithm was constructed to determine the dispersion characteristics. Elements of the dispersion equation consists of special Hankel functions of the 1st and 2nd kind of the n-th order. As is known, Hankel functions are expressed in terms of Bessel functions of the first and second kind of the nth order $H_п^{(1),(2)}{}_{(кr)} = I_{п\,(кr)} \pm iY_{п(кr)}$.

The elements of the dispersion equation are expressed in terms of the special Bessel and Hankel functions of the first and second kind of the n-th order. Now let us consider the computations of these functions on a computer. The Bessel and Neumann functions of the nth order can be determined by an infinite series:

$$J_n(z) = \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{1}{2}z\right)^{n+2k}}{k!\,\Gamma(n+k+1)}$$

$$Y_n(z) = \left\{\frac{2}{\pi} J_n(z)\left(\gamma + l^n \frac{z}{2}\right)\right\} - \frac{1}{\pi}\sum_{k=0}^{k-1} \frac{(n-k-1)}{k!}\frac{z^{2k-n}}{2} +$$

$$+\frac{1}{\pi}\sum_{k=0}^{\infty 1} \frac{(-1)^{k+1}\left(\frac{z}{2}\right)}{k!(k+n)!}\{\Phi(n)+k\},$$

Where $\gamma = 0.5772$ – Euler's constant, $\Phi(n) = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$ $u\ \Phi(0) = 0$.

If n = 0 and 1 are known, we can calculate the Bessel and Neumann functions of any order from the following recurrence relations (F_n=J_n;Y_n):

$F_{n+1}(z) = \frac{2\pi}{z} F_n(z) - F_{n-1}(z)$, где z – complex quantity.

Complex number z = x+iy can be represented in the form $z = \rho e^{i\phi}; \rho = \sqrt{x^2+y^2}, \phi = arctg\frac{y}{x}$.

From the relation we obtain:

$$J_0(\rho e^{i\phi}) = \sum_{k=o}^{\infty}(-1)^k \frac{\left(\frac{\rho}{2}\right)^{2k}}{(k!)^2} e^{2k\phi} = U_0(\rho,\phi) + iV_0(\rho,\phi);$$

$$U_0(\rho,\phi) = \sum_{k=o}^{\infty}(-1)^k \frac{\left(\frac{\rho}{2}\right)^{2k}}{(k!)^2}\cos 2k\phi = U_0(\rho,\phi)$$

$$V_0(\rho,\phi) = \sum_{k=o}^{\infty}(-1)^k \frac{\left(\frac{\rho}{2}\right)^{2k}}{(k!)^2}\sin 2k\phi = U_0(\rho,\phi).$$

For the series above, the remainder does not exceed the first discarded term. If you select U_0(ρ, ϕ) and V_0(ρ, ϕ) by 26 series members (polynomials of degree 50 with respect to r), then the error in modulus will be less than $1.04\left(\frac{p}{2}\right)^{52}\frac{1}{(26!)^2}$, the maximum value of which (for ρ <10) is approximately equal to $1.5 \cdot 10^{-17}$.

To calculate the frequencies, as well as the attenuation coefficients, a program was developed in the ++ CI language. The roots of equation (7), which are functions of the parameter ξ , were determined by the Mueller method according to the formula

$$\xi_l = \xi_{l-1} - \left.\frac{\Delta(\omega,\xi)}{\partial\Delta(\omega,\xi)/\partial\xi}\right|_{\xi=\xi_{l-1}} \qquad (l = 1,2,\ldots),$$

the process of obtaining iterations was terminated when inequality $|\xi_l - \xi_{l-1}| \le \varepsilon,$

where ε – an arbitrary sufficiently small number. As the zeroth approximation, at the first stage of the computations, the solutions of the equation were chosen for sufficiently small ε.

Numerical results of natural oscillations of layered bodies

We denote by (4.7b) $\lambda = -\omega^2$,and reversing the matrix $[M]$, we pass from (7) to the standard Eigen value problem: $[R]\{A\} = \lambda\{A\}, [R] = \lambda[M]^{-1}[PK]$.

By the frequencies found ω^2 , the Eigen vectors $\{A\}$.

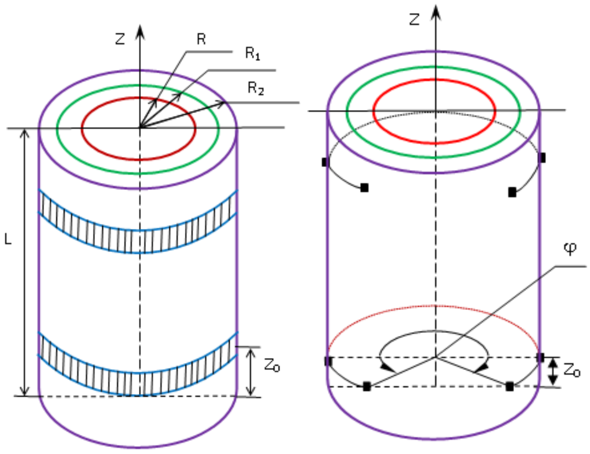

Fig2. *Calculation scheme of a double-layer cylinder*

If the lamellar or cylindrical layered bodies are connected to the bases (or elastic medium) through the models of the Wine cler, then the arising forces assume the following form (without taking into account the inertial inertia)

$$q_r = k_0 w \qquad (8)$$

We consider the vibrations of a three-layer plate connected with the elastic inertial base of the Vin Kler:

$$q_r = k_0 w + m_f \ddot{w}, \qquad (9)$$

where m_f - mass inertial coefficient of elastic resistance.

If the viscous properties of the interaction between the system and the base are taken into account, then (9) takes the following form:

$$q_r = m_f \frac{\partial^2 w}{\partial t^2} + k_{00}\left[w - \int_{-\infty}^{t} R_k(t-\tau)w(\tau)d\tau\right] . \qquad (10)$$

Unlike the Vin Kler model, the Pasternak inertial model takes into account the shear resistance in the external environment:

$$q_r = k_0 w + m_f \ddot{w} + \tau_f \Delta w \ , \qquad (11)$$

where τ_f - shear rate, Δ - the Laplace operator. If the viscous properties of the interaction between the system and the base are taken into account, then (11) takes the following form:

$$q_r = m_f \frac{\partial^2 w}{\partial t^2} + k_{00}\left[w - \int_{-\infty}^{t} R_k(t-\tau)w(\tau)d\tau\right] + \tau_{ff}\left[\Delta w - \int_{-\infty}^{t} R_f(t-\tau)\Delta w(\tau)d\tau\right] , \qquad (12)$$

where k_{00}, τ_{ff} - instantaneous compression and shear ratios; R_k, R_f - respectively, the relaxation nuclei.

Numerical results are obtained for a freely supported circular three-layer cylindrical shell (D16T-fluoroplastic) with parameters:

$h_1 = h_2 = 0.025; c = 0.02; R = 1$, located in the inertial environment of the Wine Clerus (Figure 3). As the relaxation nucleus of a viscoelastic material, we take a three-parameter core $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$ Rizhanitsena-Koltunova [12], which has a weak singularity, where A, α, β - material parameters [12]. We take the following parameters:

$A = 0{,}048; \quad \beta = 0{,}05; \quad \alpha = 0{,}1$, using the complex representation for the elastic modulus described earlier. The roots of the frequency equation are found by the Mueller method, at each iteration of the Muller method the Gauss method is used with the separation of the main element.

Thus, the solution of equation (7) does not require the disclosure of the determinant. As the initial approximation, we choose the natural frequencies of the elastic system. In the figures 3 and 4 ($-L = 1.5R;\ L = 9R$) the change in the real ($\omega_{R,k}$) and imaginary ($\omega_{I,k}$) (k=1,2,3) parts of the complex natural frequency depending on the rigidity coefficient κ_0 viscoelastic medium. The presence of an external medium practically does not affect the frequencies of torsional oscillations ω_{012}.

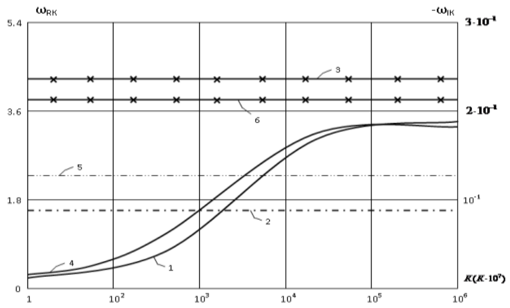

Fig3. *Change in real parts of complex frequencies depending on the interaction coefficient.*

- $\omega_{R,11.}$ **2.** $\omega_{R,02.}$ **3.** $\omega_{R,03.}$ **4.** $\omega_{I,11.}$ **5.** $\omega_{I,02.}$ **6.** $\omega_{I,03}$

With decreasing rigidity, the frequency ω_{011} y decreases to zero. Frequency of torsional oscillations ω_{013} is limited (constant), for long and hard environments it increases sharply. The results of calculations for elastic mechanical systems, when the rheological properties of materials are not taken into account, the elements of the mechanical system are compared with the results obtained by E. Starovoitov [13, 14]. Results differ with a difference of up to 10%.

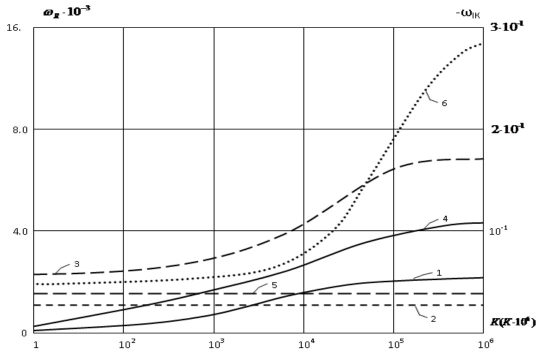

Fig4. *Change in real parts of complex frequencies as a function of the interaction coefficient.*

- $\omega_{R,11.}$ **2.** $\omega_{R,02.}$ **3.** $\omega_{R,03.}$ **4.** $\omega_{I,11.}$ **5.** $\omega_{I,02.}$ **6.** $\omega_{I,03}$

Figures 5 and 6 show the change in real and imaginary parts of frequencies depending on the thickness of the core (middle layer) for different lengths of a three-layer cylinder $L = 2.5R;\ L = 7.5R$. It can be seen from the figure that with an increase in the Winkler interaction coefficient within $10 \le k_0 \le 10^6$ The real and imaginary parts of the eigenfrequencies increase monotonically. Small value $0 \le k_0 \le 10$ the interaction coefficient of the Vin Cler almost does not affect the behavior of the natural frequencies.

It can be seen from the figures that as the thickness of the aggregate increases, the corresponding frequencies increase and approach asymptotics. The behavior of the real and imaginary parts of the frequency is almost the same. The natural frequencies (real and imaginary parts) when accounting for the Winkler base increase almost 20%. The account of inertial terms reduces the frequencies depending on the m_f , With an increase in the inertia force of the frequency decreases and approaches the asymptote.

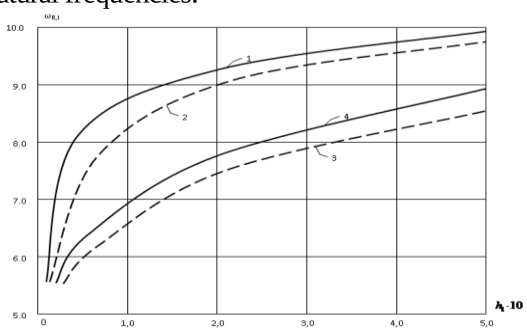

Fig5. *Change of real and imaginary parts of the natural frequencies, depending on the thickness of the middle (filler) layer.*

- real frequency parts, for Winkler inertial-free media ($\omega_{Rm}10$);
- real parts of the frequency, for inertial-free media of Pasternak ($\omega_{Rm}10$);
- imaginary parts of the frequency, for the inertial-free Winkler media ($\omega_{Im}10^{-2}$);
- imaginary parts of the frequency, for the inertial-free media of Pasternak ($\omega_{Im}10^{-2}$);

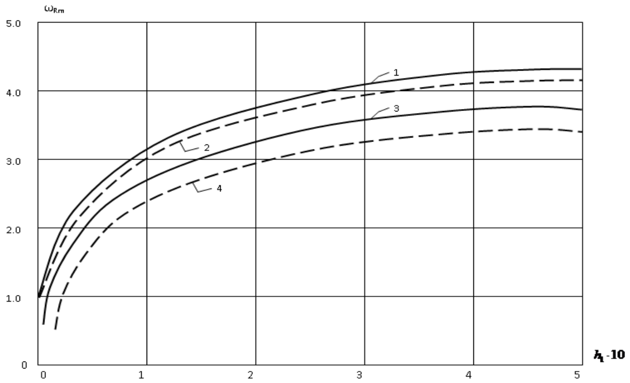

Fig6. *Change of real and imaginary parts of the natural frequencies, depending on the thickness of the average (filler) layer.*

- real parts of the frequency, for inertial corresponding Winkler media ($\omega_{Rm}10$);
- real parts of the frequency, for the inertial corresponding media of Pasternak ($\omega_{Rm}10$);
- maginary parts of the frequency, for inertial corresponding Winkler media ($\omega_{Im}10^{-2}$);
- imaginary parts of the frequency, for the inertial corresponding media of Pasternak ($\omega_{Im}10$);

An increase in the thickness of the aggregate causes an increase in the real and imaginary part of the complex frequencies of the shell three-layer systems. At the same time, the results for the corresponding models of Winkler and Pasternak (both with and without inertia of the external environment) practically coincide. Table 1 shows for each index m four frequencies ω_{mp} ($p = 1.....4,\ k_0 = 0,\ L = 1{,}5R,\ 9R$).

Table1: *Real parts of the frequencies of the three-layer shell*

p/m	L=2R			L=10R		
	0	1	2		1	2
1	0	2.822	3.021	0	2.838	2.774
2	3.602	6.068	11.433	3.602	3.655	7.859
3	4.478	7.233	12.203	4.478	6.621	11.024
4	5.295	8.296	15.993	5.295	7.958	14.959

Let us consider two variants of the dissipative system. In the first variant, a homogeneous system is considered, i.e. all the rheological properties of all layers (elements of the mechanical system) are the same (Figure 7).

The frequency dependence of k0 turned out to be the same as for a dissipative homogeneous system: the corresponding curves coincide with an accuracy of up to 5%. As for the coefficients of damping, their behavior has changed radically: the dependence $\omega_I \sim \xi\ (\kappa_0, l$ and others) became nonmonotonic (Figure 8).

Of particular interest for practice is the minimum value of the damping factor (for fixing ξ) [15,16]

$\delta_\omega = \min(-\omega_{Ik})$. (13)

Value δ_ω determines the damping properties as a whole. In the case of a homogeneous system δ (we shall call it the global damping coefficient) is determined by the imaginary part of the first complex natural frequency modulo.

In the case of an inhomogeneous system, the imaginary parts of both the first and second frequencies act as the global damping coefficient. "Change of roles" occurs with a characteristic value ξ; while the real parts of the first and second frequencies are the closest. The global damping factor at the specified characteristic value ξ has a pronounced maximum.

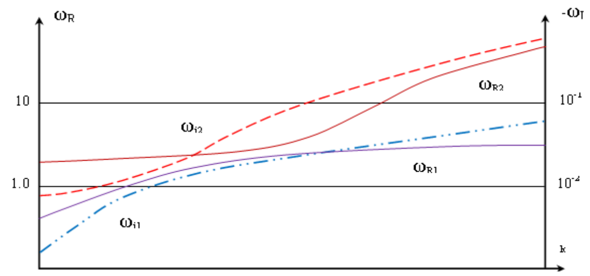

Fig7. *Changing the proper complex frequencies from the wave number, for dissipatively homogeneous system.*

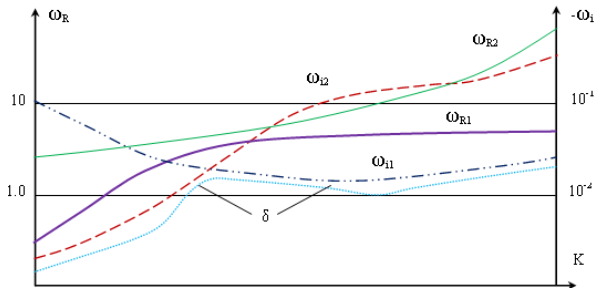

Fig8. *Changing the proper complex frequencies from the wave number, for a dissipatively inhomogeneous system.*

The change in the parameter, on which the global coefficient of damping depends so substantially, can be achieved by varying the geometric dimensions or physical properties, thereby opening up the perspective possibility of effectively controlling the damping characteristics of dissipatively inhomogeneous viscoelastic systems. Such results were obtained both for the Koltunov-Rzhanitsin relaxation nucleus and for the fractional exponential core of Rabotnov. In (1) and (2) the following notation is adopted: $\sigma_{ij}^{(n)}$ — components of the stress tensor, $\vartheta_i^{(n)}$ — displacement vector components,

$$\tilde{E}_n = E_n\left(1-\Gamma_n^{\bullet}\right); \quad \tilde{\nu}_n = \nu_n + \frac{1-2\nu_n}{2}\Gamma_n^{\bullet};$$

$$\Gamma_n^{\bullet} f(t) = m_n \int_{-\infty}^{t} \Im_{-1/2}^{(n)}(-\beta_n, t-\tau) f(\tau) d\tau \quad (14)$$

Here E, ν – instantaneous values of Young's modulus and Poisson's ratio, m_n, β_n— parameters of the material. As the kernel of the integral operator we use the fractional exponential function of Rabotnov [17]

$$m_n \Im_{-1/2}^{(n)}(-\beta,t) = m_n t^{-1/2} \sum_{j=0}^{\infty} \frac{(-\beta_n)^j t^{j/2}}{\Gamma[(j+1)/2]},$$

where $\Gamma(j) = \int_0^\infty y^{j-1}\exp(-y)dy$ -gamma function.

In studying the natural oscillations, we will investigate the properties of those modes (the modes are understood to mean particular solutions of the equations of motion in displacements that satisfy homogeneous boundary conditions on the face surfaces), which vary in time according to the harmonic law and satisfy the equations of motion (1), the equations of state (2) and homogeneous boundary conditions on the front surfaces. The results of calculations in low frequency regions differ up to 15%, and in high frequency regions - up to 60%. This difference is explained by the fact that, in the low-frequency region, the change in Poisson's coefficients, depending on the material rheology, can be neglected. And in high frequency regions, variation of Poisson's coefficients, depending on the material rheology, cannot be neglected.

Forced oscillations

The system of differential equations of motion in the displacements (1) with the inertial basis of the Vin cler takes the following form

$$L_2(a_1u + a_2\psi - a_3w_{,r}) = 0, \quad L_2(a_2u + a_4\psi - a_5w_{,r}) = 0,$$
$$L_3(a_3u + a_5\psi - a_6w_{,r}) - M_0^{*}\ddot{w} - k_0w = -q, \quad (15)$$

where $M^{\bullet} = M_0 + m_f$ - specific weight of the plate and base. The system of differential equations of motion in the displacements (1) with the inertial base of Pasternak takes the following form:

$$L_2(a_1u + a_2\psi - a_3w_{,r}) = 0,$$
$$L_2(a_2u + a_4\psi - a_5w_{,r}) = 0, \quad (16)$$
$$L_3(a_3u + a_5\psi - a_6w_{,r}) + t_f\Delta w - M_0^{*}\ddot{w} - k_0w = -q,$$

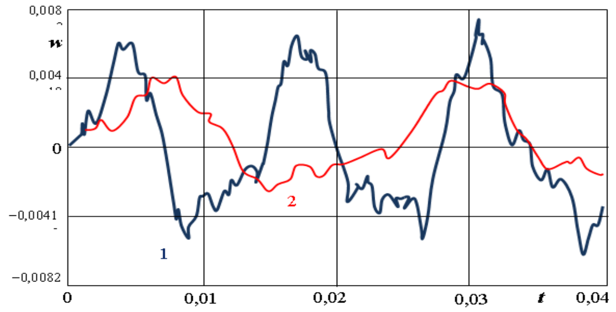

Fig9. *Change in deflection as a function of time: 1- non-inertial base; 2-inertial base.*

The solutions of systems (15) and (16) are taken in the form of convergent expansions in a series in systems of eigenfunctions:

$$W(r,t)=\sum_{n=0}^{\infty} v_n^* T_n(t), \quad \psi(r,t)=b_2\sum_{n=0}^{\infty}\varphi_n^* T_n(t), \qquad (17)$$

$$u(r,t)=b_1\sum_{n=0}^{\infty}\varphi_n^* T_n(t), \quad q(r,t)=M_0^*\sum_{n=0}^{\infty} v_n^* q_n(t),$$

where $T_n(t)$ is taken in the form:

$$T_n(t)=A_n\cos(\omega_n t)+B_n\sin(\omega_n t)+\frac{1}{\omega_n}\int_0^t \sin(\omega_n(t-\tau))q_n(\tau)d\tau$$

The change in the deflection as a function of time, of a three-layer system clamped along the contour fixed to the base of the Vin cler, under the influence of vertical harmonic loads is shown in Fig. 10.

$$q(r,t)=q_0(D\cos(\omega_k t)+E\sin(\omega_k t)), \quad (D,E-const). \qquad (18)$$

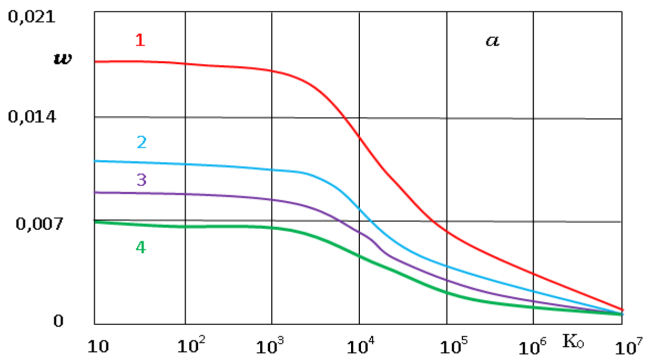

Fig10. *Change in the maximum deflection (a) and shear (b) on the basis of Pasternak under impulse loading.*

Figure 9 shows: 1- inertial-free base; 2-inertial base. When obtaining numerical values, the parameters (18) are assumed to be equal to unity. Numerical results show that the inertia of the base reduces the maximum vertical displacement (deflection) to 38%, and taking into account the rheological properties of the "structure-base" interaction, up to 44%. Now consider the oscillations of the structures with the inertial base Pasternak (Figure 10).

Figure 11 shows the change in the deflection in the center of the plate without taking into account the elasticity of the base. For the first curve $t=002,\ k_0=10^3$. For the second curve $t=0.03,\ k_0=10^4$.

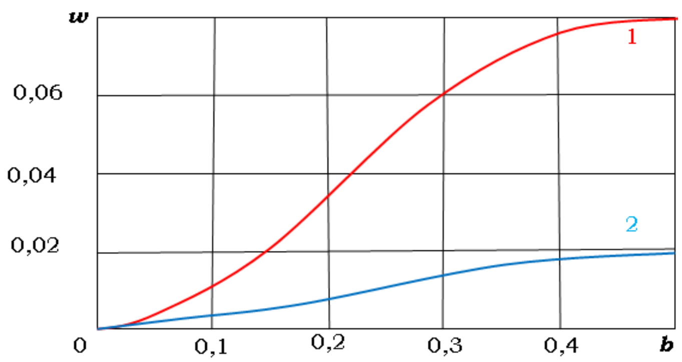

Figure11. *Change in deflection in the center of the plate without taking into account the elasticity of the base.*

1- $t=0.02,\ k_0=10^3$. *2-* $t=0.03,\ k_0=10^4$

Table 2 compares the results of the first three complex of the vibration of a two-layered cylindrical body using a package of the developed algorithm [18]. It can be seen that the results of calculations differ to 20%.

Damping of resonance oscillations of rod and tube oscillations

To reduce the resonant vibrations, in addition to the methods discussed above, layers made of vibrating absorbing materials are also used (Figure 12). In this case, resonance oscillations can be reduced in a wide range of frequencies with an insignificant increase in the masses and overall dimensions of the structures. The effectiveness of the use of vibration absorbing materials is illustrated in Figure 12. The use of a board consisting of three layers of glass fiber reinforced plastic with squeezed thin layers of vibration-absorbing material (VP) sandwiched between them reduced resonant vibrations by a factor of 3 [19].

Table 2. *A comparison of the results of the first three complex of the oscillation of a two-layer cylindrical body is used, using a package of the developed algorithm and the FEM.*

Z_0^*	ω_R^*			ω_I^*		
	1	**2**	**3**	**1**	**2**	**3**
0,0	0.991 / 0.961	1.649 / 1.636	1.810 / 1.728	0.041 / 0.037	0.040 / 0.040	0.081 / 0.073
0,2	1.129 / 1.089	1.831 / 1.809	1.997 / 1.909	0.050 / 0.046	0.049 / 0.051	0.115 / 0.107
0,4	1.278 / 1.239	2.037 / 2.024	2.111 / 2.042	0.058 / 0.055	0.060 / 0.060	0.136 / 0.130
0,6	1.459 / 1.420	2.235 / 2.156	2.285 / 2.180	0.069 / 0.065	0.160 / 0.181	0.159 / 0.153
0,8	1.688 / 1.611	2.285 / 2.132	2.289 / 2.168	0.083 / 0.076	0.133 / 0.119	0.135 / 0.129
1,0	1.965 / 1.760	1.906 / 1.816	1.994 / 1.949	0.097 / 0.082	0.101 / 0.093	0.108 / 0.111
1,2	1.610 / 1.542	1.611 / 1.546	2.293 / 2.155	0.078 / 0.072	0.079 / 0.074	0.185 / 0.180

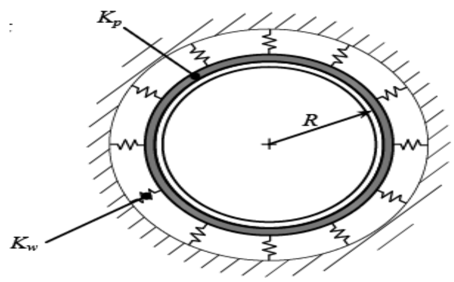

Fig12. *The calculation scheme.*

Figure 12 shows examples of the use of vibration absorbing materials to suppress resonance oscillations in structures. We seek the solution of the boundary value problem in the form: $q_*(t) = \vartheta_k(x)e^{-i\omega t}$, where $\vartheta_k(x)$ - complex waveform: $\omega = \omega_R + i\omega_I$ - the required complex frequency. The problem reduces to solving a homogeneous algebraic equation:

$$|\Delta(\omega_R, \omega_I) = 0|. \qquad (19)$$

For the calculations, the following values of the parameters of the problem:

$$\frac{L}{R} = 4;\ \frac{R_1}{R} = 0{,}3;\ \frac{R_2}{R} = 1.01;\ R_C^{(1)} = 0;\ R_S^{(1)} = 0{,}2;\ F_C^{(1)} = F_S^{(1)} = 0;$$

$$\frac{B_0^{(1)}}{G_0^{(1)}} = 24.7;\ \frac{G^{(2)}}{G_0^{(1)}} = 10^2;\ \frac{B^{(2)}}{G_0^{(1)}} = 217.0;\ \frac{\rho^{(2)}}{\rho^{(1)}} = 1$$

and it was also assumed that the filler material behaves in an elastic manner during volume deformation.

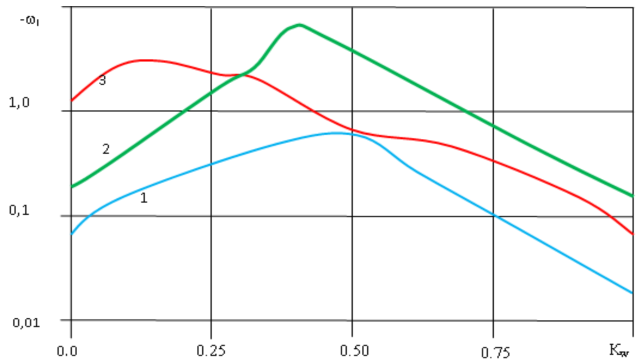

Fig13. *Dependence of the imaginary parts of the vibration frequencies of a two-layer cylinder on the interaction coefficient Kw.*

- **-ω_{I1}; 2.- ω_{I2} ; 3.- ω_{I3}.**

It can be seen from Fig. 13 that for a dissipative inhomogeneous system the imaginary parts of the complex frequency do not change monotonically depending on the interaction coefficient, for a dissipative homogeneous system such a dependence does not take place.

A similar dependence is shown in the figure 14.

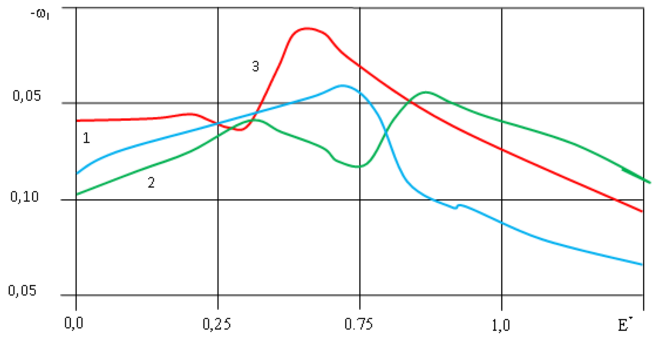

Fig14. *Dependence of the imaginary parts of the natural oscillation frequencies of a two-layer cylinder on the ratio of the modulus of elasticity of the filler to the elastic moduli of the shell.*

- **-ω_{I1}; 2.- ω_{I2} ; 3.- ω_{I3}.**

At the decision of a problem both of the above-stated variants of use of package MAPLE were applied (figure 13 and drawing 14).

Conclusions

- A scientifically substantiated methodology and algorithm for dissipative mechanical systems consisting of layered plates and shells is developed. As an example, we consider a three-layer construction with elastic (viscoelastic) and inertial-free media of Wine Cler and Pasternak, with external dynamic effects.
- In solving the problems of intrinsic and forced oscillations of a dissipative heterogeneous three-layer structure, some general laws for the natural frequencies and damping indices are found. A method is developed for calculating vibro protective devices, considered as elastically viscous hereditary systems. The method showed good agreement between the theoretical and experimental data and allowed to predict the parameters of the damping structures.
- The analysis of theoretical and experimental amplitude-frequency response (amplitude-frequency characteristic) showed satisfactory convergence of calculation and experiment: the frequency error did not exceed 15%, amplitude -28%.
- It is established that the presence of rubber shock absorbers reduces the oscillation amplitude of the equipment by up to 30%.

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