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Compressive Sensing in the DFrFT Domain

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ABSTRACT

In this study, linear frequency modulated (LFM) signals are reconstructed by compressive sensing (CS) making the signals sparse in appropriate discrete fractional Fourier transform (DFrFT) domains. We first transform an LFM signal into several sparse structures using DFrFT matrix, and select the optimum order in which the signal is sparsest structure. Then, we employ CS by taking a few random measurements. CS simulations show that we successfully reconstruct the signal and obtain the minimum mean square error (MSE) in the optimum DFrFT order.

Keywords: Discrete fractional Fourier transform, compressive sensing

INTRODUCTION

Most of the signals are sparse or compressible so they have an appropriate transform domain with a small number of sparse coefficients. Thus, these signals can be made sparse by expressing them as linear sums of basis functions, such as sinusoidals, wavelets, canonical basis, and other basis. An important point of the compressive sensing (CS) method is to obtain the measurements from the projection region defined as compressed region. Then reconstruction is provided by optimization. The sparse structure, measurement number, optimization method are important sub-sections in order to reconstruct the signals by CS [1-5].

In the recent years, CS via fractional Fourier transform (FrFT) domains is very interesting. In [6], the calculation of the discrete fractional Fourier transform (DFrFT) matrix differs from the calculation of the DFrFT matrix used in this study and the computation of DFrFT is achieved by sampling FrFT. At another studies [7, 8], where some parts of the signal is missing or incomplete, it has been supposed that a prior information about the signal is at hand, and it is tried to recover signals by recurrent algorithms, such as projection onto convex sets.

In this study the DFrFT matrix derived from [9] is employed. The linear frequency modulated (LFM) signals are reconstructed by CS in optimum DFrFT order. The simulation figures and mean square errors (MSE) table are given. The study is concluded in last part and determined that the CS with optimum ordered-DFrFT matrix can be efficiently used for LFM-type signals.

COMPRESSIVE SENSING

n - length LFM-type signal x with k-sparse coefficients is given as

$$x = \Psi s \qquad (1)$$

where Ψ is the basis matrix.

If m measurements are taken from random projections onto Φ which is projection matrix as follows

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$$y = \Phi x = \Phi \Psi s \tag{2}$$

In this study x is the linear sum of the DFrFT basis.

$$y = \Phi W_n^a s \tag{3}$$

where W_n^a is the a-th order DFrFT matrix [9].

The choice of DFrFT order results in a better sparse representation and the more sparsity results in a better reconstruction and thus better CS is obtained.

We transform a wide-band LFM-type signal into a narrow-band signal in the DFrFT domain with appropriate order to obtain the sparse coefficients for use in CS. We apply CS, in which we take random quarter quantity of the whole. As a result of the convex optimization [10], we reconstruct the signals, in which the original and reconstructed signals are shown in Figure 1-6.

The deformation between the original and reconstructed signals can be calculated by the mean square error as follows:

$$MSE = \frac{1}{n} \sum \left(x_{original} - x_{reconstructed} \right)^2 \tag{4}$$

SIMULATIONS

A 128 length LFM-type signal that is highly dense in time domain is transformed into discrete fractional Fourier domain to form the sparse structure. In order to examine the effect of the DFrFT order, 0.1, 0.3, 0.5, 0.7, 0.9 and 1.1 order values are investigated, in which the optimum DFrFT order is obtained to be 0.7 to get the signal in its most sparse structure. The original and the reconstructed signals are plotted together for a better illustration.

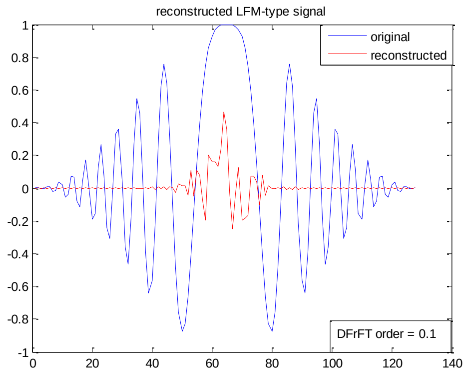


Figure1. *CS with DFrFT order=0.1*

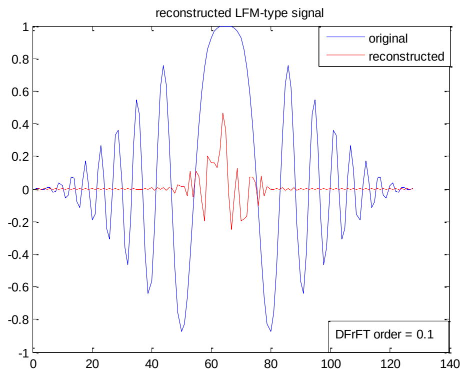


Figure2. *CS with DFrFT order=0.3*

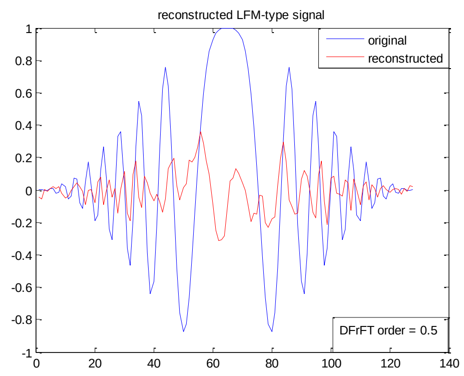


Figure3. *CS with DFrFT order=0.5*

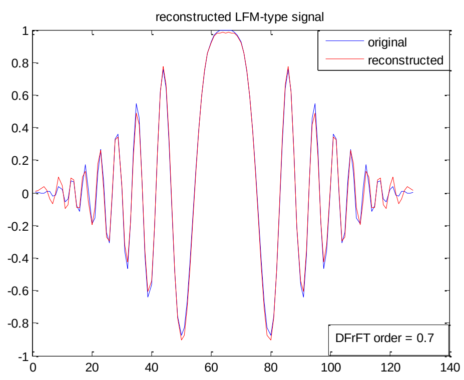
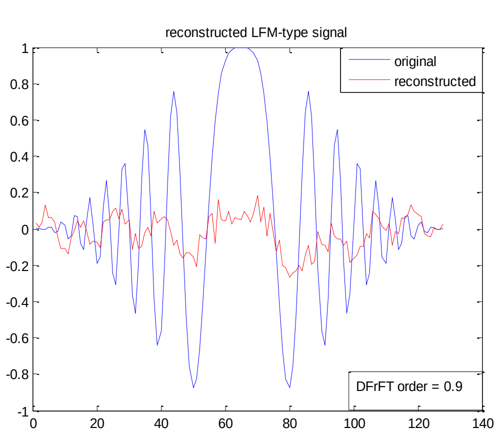


Figure4. *CS with DFrFT order=0.7*



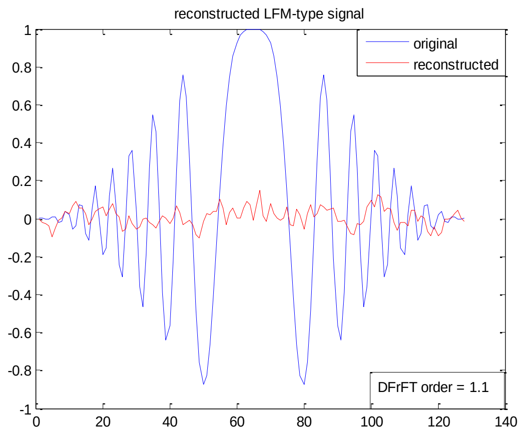


Figure5. *CS with DFrFT order=0.9* **Figure6.** *CS with DFrFT order=1.1*

MSE between the original and reconstructed signals for each figures are given in Table 1.

Table1. *The MSE of CS*

DFrFT order	MSE
0.1	0.2203
0.3	0.2288
0.5	0.2082
0.7	0,0009
0.9	0.1891
1.1	0.1767

CONCLUSION

LFM-type signals are not sparse in time domain. However, they are sparse in optimum DFrFT domain. In this work, we propose CS, in which we preprocess the LFM-type signals decomposing it sparsely with optimum-order DFrFT. Simulation results show that minimum MSE error is obtained in the optimum DFrFT order. This method is advantageous for saving and storing information effectively in radar, sonar, communication systems where LFM-type signals are employed.

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