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Analysis of Longitudinal Slots in Rectangular Waveguide Using Finite Difference Method

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ABSTRACT

Recent advances in computer speed and storage have led to an increasing interest in developing new methodologies to satisfy a need for accurate and efficient numerical computation. The use of the Finite difference Method (FDM) for the numerical solution of electromagnetic scattering in unbounded regions requires proper absorbing boundary conditions (ABC) on the outer surface that truncates the infinite three-dimensional space. In this work we analyze a single radiating longitudinal slot in the broad wall of a rectangular waveguide using FDM which had been previously studied with the Method of Moments (MoM). It leads to a sparse matrix however its size becomes extremely large

INTRODUCTION

Since the evolution of computers in early 60's, the numerical solution of equations that describe physical phenomena opened new horizons in our ability to better understand the behavior of nature. This ignited a tremendous effort among scientists and engineers in developing computer-aided methods that would predict the behavior of such physical phenomena. The new era of computer simulation was born.

In the world of electromagnetic, computer simulation techniques have proven to be powerful tools in predicting and giving a better understanding of the behavior of electromagnetic fields and the performance of various devices. For scattering problems in the frequency domain, where the operating frequency is known, integral equation techniques, such as the Method of Moments (MoM), were the first to be exploited and for years they dominated the research, as well as the commercial market [1]. Such techniques necessitate the use of green's functions either scalar or dyadic. More on the available computer programs based on integral equations and moment method solutions may be found in [2], chapter 12.

The implementation of numerical techniques generally leads to a system of equations, which in matrix notation is:

$$[A].[X] = [b] \quad (1.1)$$

$[A]$ Is a square matrix $[b]$ is the known right hand side, usually called the excitation and the solution is the unknown column vector$[X]$.

For certain type of problems, where the spatial domain is bounded by perfectly conducting walls differential equation techniques, such as Finite Differences (FD) or Finite Elements (FE), are superior to integral equations as they are easier to formulate without boundary conditions (and slightly more complicated when BC's are included). They are based on discretization of the partial differential equation. [A] The final matrix they produce is sparse. After a standard procedure the matrix can be sometimes made banded. Problems with many dielectric materials are handled without extra computational cost. FD are more suitable for problems with more regular geometries, while FE can easily handle any kind of arbitrary geometries and give better accuracy when highly complex in homogeneities are present.

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Both techniques, FD and FE, have been exploited in the past also for radiation problems from wire or dielectric scatterers. For such open boundary problems though, where the geometric domain is not bounded by any surface with known absorbing boundary conditions and extends to infinity FD or FE suffers because artificial absorbing boundary condition (ABC) has to be used in order to truncate the infinite domain.

PROBLEM DEFINITION

This thesis deals with the implementation of the Finite Difference Method (FDM) for solving open boundary vector wave problems. An ABC of the first order [3] is introduced. It is based on a local differential operator, in other words it does destroy the sparsity and symmetry of the FD matrices to a large extent. This boundary condition is applied on the surface of a closed box. The mathematical box truncates the infinite domain of the problem to a finite one and should completely enclose the volume of interest. The volume of interest is defined as the three-dimensional region that contains all the metallic and dielectric scatterers, which may be of any shape and complexity. The FDM can be applied in the finite region.

The role of this new type of boundary condition is at absorb all outgoing electro-magnetic waves causing almost no reflection on the surface of the box. Due to its absorbing character, the boundary condition is called an Absorbing Boundary Condition (ABC) and the Surface where it occurs the absorbing boundary surface (ABS).

PRELIMINARY STUDIES

Analysis of longitudinal slot in rectangular waveguide geometries and rectangular structures has made a significant progress; however, the development is relatively slow with other slot geometries. The MoM formulations are contained in [4-6]. The start of the formulation in terms of unknown magnetic current can be made from text book like that of Markov [7]. The finite element problem can be studied from ref. [3]. The equations are formed from the curl equations of Maxwell and take the form$\nabla \times \nabla \times \vec{E} - k^2\vec{E} = 0$. Details are given in chapter 2. The boundary conditions to be satisfied are that the tangential components of the electric field should be forced to go to zero on the metallic boundaries. The conditions at the input and matched end of the waveguide are also discussed in chapter 2. The ABC for this particular problem (of slot) is given in chapter 3 in the component form making use of the Wilcox expansion [8]. The full geometry is shown in the figure 1 below. It is a short section of a rectangular waveguide and connected to a matched termination on pink side. At the metallic wave guide walls the tangential components of the electric fields will be zero. The excitation is from an incident TE_{10} wave in the waveguide. We have already stated that all differential operators are discretized as finite differences. In this work try as far as possible to use central difference formulas to preserve the symmetry of the matrix [A] in Eq. (1.1). However, when the boundary conditions contain derivatives we are forced to use either forward or backward difference formulas which disturbs the symmetry. In the case of an ABC the sparsity is also reduced because of the presence of large number of terms. The non zero terms in vector [b] Eq. (1.1) is obtained from the non zero term in the R.H.S of the boundary condition at the input port that is depicted by orange lines in figure 1. It will be discussed in more detail.

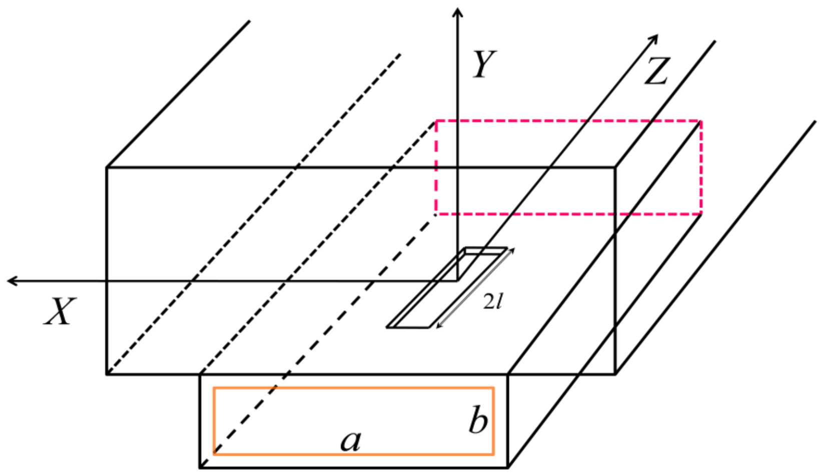


Figure1. *Longitudinal slots in Rectangular waveguide*

METHODOLOGY AND THESIS ORGANIZATION

The problem is divided into two sections that is the region, inside the waveguide and the region bounded by the ABC and the ground plane. The two are joined by the slot opening where the continuity of the fields is enforced. The different part of the matrix [A] is contributed by the different the different regions will be discussed later. The size of the matrix [A] allows only an iterative solution of Eq. (1.1) and we use the method of conjugate gradient for this purpose. Chapter 4 contains the algorithm for forming the matrix [A] from the contributions of the section inside the waveguide and that bounded by ABS. The conjugate gradient algorithm for solving Eq. (1.1) along with how the real computation proceeds is outlined. In chapter 5, all computer programs and the results are given for some of the computed elements of [A] along with the other relevant quantities.

DERIVATIONS OF THE WAVE EQUATIONS ALONG WITH BOUNDARY CONDITIONS

In the free space the Maxwell's equations are

$\nabla\times\vec{E} = -j\omega\mu_0\vec{H}$ $\nabla\times\vec{H} = j\omega \in_0 \vec{E}, \nabla.\vec{E} = 0, \nabla.\vec{H} = 0$ Since the interior of the waveguide and the region enclosed by the ABC and the ground plane (the radiation region proper) is free space, these are the appropriate starting equations. The coordinate system used is as shown in the fig. 1. Taking the curl of the first of the equations, we get

$$\nabla\times\nabla\times\vec{E} - k^2\vec{E} = 0 \tag{2.1}$$

Where $k^2 = \omega^2\mu_0 \in_0$. One can easily verify that divergence equation is satisfied. The boundary conditions are $\vec{E}_{tan} = 0$ on all the metallic surfaces and the derivative $\frac{\partial\vec{E}_{norm}}{\partial n} = 0$ where n is the coordinate normal to the surface. This last condition on the normal component $\vec{E}_{norm}$ is a consequence of $\nabla.\vec{E} = 0$, at the metal surface. The conditions at the input and matched end of the waveguide are obtained under the assumption of presence of only the dominant TE_{10} mode at these planes which is

$\frac{\partial E_{total}}{\partial z} = \sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}(-j\beta) + \Gamma\left[\sin\left(\frac{\pi x}{a}\right)e^{+j\beta z}(j\beta)\right]$ approximately true if the planes are sufficiently far removed from the slot where all the mode have decayed. The detailed derivation is as follows,

For $z < 0$

Incident field $$\vec{E}_{inc} = \sin(\pi x/a)e^{-j\beta z} \tag{2.2}$$

Reflected wave $$\vec{E}_{ref} = \Gamma\left[\sin(\pi x/a)e^{+j\beta z}\right] \tag{2.3}$$

Addition of the above two equations (2.2) and (2.3). The total field is at z=-4L is

$$\vec{E}_{total} = \sin(\pi x/a)e^{-j\beta z}\,\hat{\boldsymbol{u}}_y + \Gamma\left[\sin(\pi x/a)e^{+j\beta z}\right]\,\hat{\boldsymbol{u}}_y \tag{2.4}$$

Differentiating this we obtain

$$\frac{\partial\vec{E}_{total}}{\partial z}\times\frac{1}{j\beta} = -\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}\hat{\boldsymbol{u}}_y + \Gamma\left[\sin(\pi x/a)\,e^{+j\beta z}\right]\hat{\boldsymbol{u}}_y \tag{2.5}$$

Substracting equations (2.5) from (2.4) we obtain

$$\vec{E}_{total} - \frac{\partial\vec{E}_{total}}{\partial z}\times\frac{1}{j\beta} = 2\sin\left(\frac{\pi x}{a}\right)e^{-j\beta z}\,\hat{\boldsymbol{u}}_y \tag{2.6}$$

Eq. (2.6) in its components form yields the following equations in x, y, z

$$E_x - \frac{\partial E_x}{\partial z}\times\frac{1}{j\beta} = 0 \tag{2.7}$$

$$E_y - \frac{\partial E_y}{\partial z}\times\frac{1}{j\beta} = 2\sin\left(\frac{\pi x}{a}\right)e^{+j\beta 4L} \tag{2.8}$$

$$E_z - \frac{\partial E_z}{\partial z} \times \frac{1}{j\beta} = 0 \tag{2.9}$$

Similarly, another boundary condition (3.27)

$$\vec{E}_{total} + \frac{\partial \vec{E}_{total}}{\partial z} \times \frac{1}{j\beta} = 0$$

For $z > 0$

Transmitted wave at z=+4L

$$\vec{E}_{trans} = T \sin(\pi x/a) e^{-j\beta z} \tag{2.10}$$

By diffentiating the Eq. (2.10)

$$\frac{1}{j\beta} \times \frac{\partial \vec{E}_{total}}{\partial z} = -T \left[\sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}\right] \tag{2.11}$$

Addition of the above two equations (2.11) and (2.10) gives us

$$\vec{E}_{total} + \frac{\partial \vec{E}_{total}}{\partial z} \times \frac{1}{j\beta} = 0 \tag{2.12}$$

Eq. (2.12) in it component form yields us the following equations

$$E_x + \frac{\partial E_x}{\partial z} \times \frac{1}{j\beta} = 0 \tag{2.13}$$

$$E_y + \frac{\partial E_y}{\partial z} \times \frac{1}{j\beta} = 0 \tag{2.14}$$

$$E_z + \frac{\partial E_z}{\partial z} \times \frac{1}{j\beta} = 0 \tag{2.15}$$

The other boundary conditions at the metal surface are

$$E_x(x,0,z) = 0 = E_x(x,b,z) \qquad E_z(x,0,z) = 0 = E_z(x,b,z) \tag{2.16}$$

$$\left.\frac{\partial E_y}{\partial y}\right|_{(x,0,z)} = 0 = \left.\frac{\partial E_y}{\partial y}\right|_{(x,b,,z)} \tag{2.17}$$

and,

$$E_y(0,y,z) = 0 = E_y(a,y,z) \qquad E_z(0,y,z) = 0 = E_z(a,y,z) \tag{2.18}$$

$$\left.\frac{\partial E_x}{\partial x}\right|_{(0,y,z)} = 0 = \left.\frac{\partial E_x}{\partial x}\right|_{(a,y,z)} \tag{2.19}$$

The Eq. (2.1) in the component form is given as

$$\frac{\partial^2 E_y}{\partial x \partial y} - \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_z}{\partial x \partial z} - \frac{\partial^2 E_x}{\partial z^2} - k^2 E_x = 0 \tag{2.20}$$

$$-\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_x}{\partial x \partial y} + \frac{\partial^2 E_z}{\partial y \partial z} - \frac{\partial^2 E_y}{\partial z^2} - k^2 E_y = 0 \tag{2.21}$$

$$-\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_x}{\partial x \partial z} - \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_y}{\partial y \partial z} - k^2 E_z = 0 \tag{2.22}$$

THE ABSORBING BOUNDARY CONDITION CONCEPT

According to the expansion theorem [8], a vector radiation function can be written as

$$\vec{E}^{sc}(r) = \frac{e^{-jkr}}{r} \sum_{n=0}^{\infty} \frac{\vec{E}_n(\theta,\varphi)}{r^n} \tag{3.1}$$

Where r, θ, φ are the spherical coordinates. The first term of this series varies as $\frac{1}{r}$ and is similar to the far field of an infinitesimal dipole. The basis of the expansion is the green's theorem.

In [3], an expansion theorem has been derived for a general mth-order differential operator B_m that exactly annihilates the first m terms of the vector expansion Eq. (3.1). Here we give an outline of that process. We now define the differential operator

$$L_m(\hat{u}) = \hat{r} \times \nabla \times \hat{u} - \left(jk + \frac{m}{r}\right)\hat{u} \tag{3.2}$$

Where $\hat{u} = \left(g\frac{\vec{A}_{nt}(\theta,\varphi)}{r^{n+1}}\right), \quad g = e^{-jkr}$ (3.3)

From (3.2), using (3.3) and for $m \geq 0$ and $n \geq 0$. We have

$$L_m\left(g\frac{\vec{A}_{nt}(\theta,\varphi)}{r^{n+1}}\right) = (n-m)g\frac{\vec{A}_{nt}(\theta,\varphi)}{r^{n+1}} \tag{3.4}$$

Also for $m \geq 0$ and $n \geq 0$

$$L_m\left[\nabla_t g\frac{A_{nr}(\theta,\varphi)}{r^{n+1}}\right] = (n+m-1)\nabla_t\left(g\frac{A_{nr}(\theta,\varphi)}{r^{n+2}}\right) \tag{3.5}$$

In both cases L_m have the effect of multiplying by $\frac{i}{r}$ (i is an integer) but leaving the $(\theta - \varphi)$ dependence unchanged. Based on these observations we can write

For m=1, 2, 3 …

$$B_m(u) = (L_{m-1})^m(u_t) + s(L_m)^{m-1}(\nabla_t)(u_r) \tag{3.6}$$

Where s is an arbitrary number, the superscript m denotes that the operator L_{m-1} is applied m times and simi1arly for the superscript $m - 1$ denotes that the operator L_m is applied $m-1$ times. Using the results from the Eq. (3.4) and Eq. (3.5) repeatedly it can be shown that

$$B_m\left(g\frac{\vec{A}_n(\theta,\varphi)}{r^{n+1}}\right) = (n+1-m)(n+2-m)\ldots(n-1)ng\frac{\vec{A}_{nt}(\theta,\varphi)}{r^{n+1+m}} + s(n+1-m)(n+2-m)\ldots(n-2)(n-2)\nabla_t g\left(\frac{\vec{A}_{nr}(\theta,\varphi)}{r^{n+m}}\right) \tag{3.7}$$

Since A_{0r} is zero, that is, radial part of the electric or magnetic field vector vanishes at a great distance from the object, which is a well known fact in electro magnetics, it can be seen that the right-hand side of Eq. (3.7) vanishes for $n = 0, 1 \ldots m - 1$, or in other words, B_m annihilates the first m terms of Eq. (3.1). Only the terms with $n > m - 1$ do not vanish but after being operated on by B_m these terms are propotional to $\frac{1}{r^{n+1+m}}$ (Note that in spherical coordinates ∇_t includes a factor $\frac{1}{r}$.) Consequently, when B_m is applied to E^{sc} as expanded in Eq. (3.1), we have

$$B_m(E^{sc}) = O(r^{-(2m+1)}) \tag{3.8}$$

Therefore, $B_m(E^{sc} = 0)$ can be regarded as an approximate absorbing boundary condition applicable to a spherical surface of radius r, which includes all sources of radiation. The same will also hold for any irregularly shaped surface which includes all sources. The first order Absorbing Boundary Conditions

$$B_1(E^{sc}) = \hat{r} \times \nabla \times E_t^{sc} - jkE_t^{sc} + s\nabla_t E_r^{sc} = 0 \tag{3.9}$$

In this thesis we take s = 1 in cartesian coordinates the components are

ALGORITHAM FOR MATRIX FORMATION AND ITS MANIPULATION

The discretized form of the wave equation in chapter 2 Eqs. (2.20) – (2.22) along with the boundary conditions Eqs. (2.7)- (2.9) , Eqs. (2.13)- (2.15) and Eqs. (2.16) - (2.19) form the submatrix $[A']$ some values of which will be given in the next chapter. Before going into the calculation of $[A']$ the program for which will be given also in the next chapter we sta

te here the discretized form of the equations stated above. Now the discretized form of Eqs. (2.20) – (2.22) is

This submatrix $[A']$ is contributed by the region inside the waveguide.

The other part of the matrix [A] formed from the contribution of the radiation region between the ground plane and ABS is $[A'']$. This follows from the same discretized wave equation (4.1)-(4.3). The boundary condition for the surface of the ground plane is similar to (4.10a)-(4.10c). On the ABS the Eqs. (3.13) – (3.15) gives us the appropriate boundary conditions which has discretized. In this thesis we have not able to complete this process. The entire matrix [A] is formed like this. Suppose

$$[A'] = \begin{bmatrix} a'_{11} & \cdots & a'_{1n} \\ \vdots & \ddots & \vdots \\ a'_{n1} & \cdots & a'_{nn} \end{bmatrix} \text{ and } [A''] = \begin{bmatrix} a''_{11} & \cdots & a''_{1m} \\ \vdots & \ddots & \vdots \\ a''_{m1} & \cdots & a''_{mm} \end{bmatrix}$$

Then [A] is formed as below

$$[A] = \begin{bmatrix} [A'] & [P] \\ [Q] & [A''] \end{bmatrix}$$

The rectangular matrix [P] and [Q] are generally all zeros except at the slot opening coordinates where the continuity of the electric field components have to be ensured. The entire matrix [A] is extremely large and the equation of the form Eq. (1.1) can be solved only by the conjugate gradient algorithm which is

$$u_k = Ap_k$$

$$\alpha_k = \bar{r}_k^T \bar{r}_k / u_k^T u_k$$

$$X_{k+1} = X_k + \alpha_k p_k$$

$$r_{k+1} = r_k - \alpha_k u_k$$

$$\bar{r}_{k+1} = A^T r_k + 1$$

$$\beta_k = \bar{r}_{k+1}^T \bar{r}_{k+1} / \bar{r}_k^T \bar{r}_k$$

$$p_{k+1} = \bar{r}_{k+1} + \beta_k p_k$$

and starting with $r_0 = b - Ax_0$, $\bar{r} = p_0 = A^T r_0$

We need not calculate all [A] at a time but only one row or column as the need arises. It slows down the computation but decreases the memory space requirement.

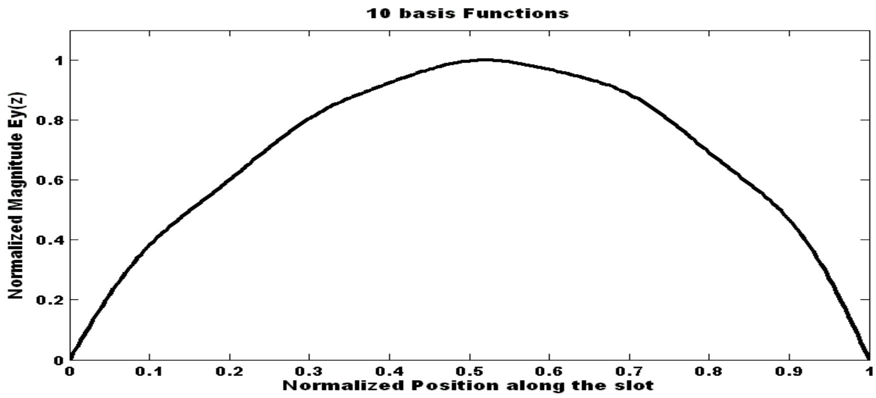
RESULTS

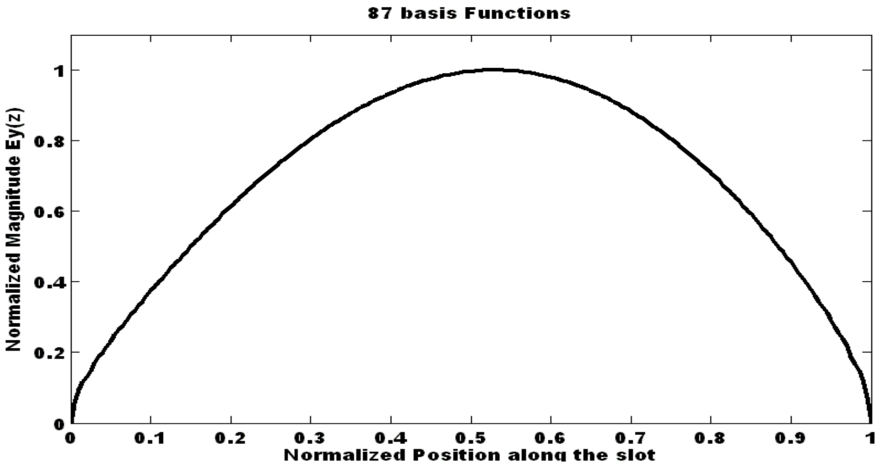
Some computed values of $[A']$ which is a matrix of size 3375×3375

Col→ Row ↓	2483	2484	2485	2486
2458	-155000.310000620	0	0	0
2459	0	-155000.310000620	0	0
2460	0	0	-155000.310000620	0
2461	0	0	0	-155000.310000620
2462	0	0	0	0
2463	0	0	0	0
2464	0	0	0	0

SCOPE OF FUTURE WORK

My senior Anu Mohammad studied the work of Josefsson [4] and confirmed his calculations. He verified the slot current distribution which is shown in Fig. 2 below. However, in Fig 3 he computed the same quantity with more number of basis function. It shows that the computations are still incomplete. Our method in this thesis if completed later will lead to better understanding of the problem





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