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Design Digital Non-Recursive FIR Filter by Using Exponential Window

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ABSTRACT

One of the most effective variables is the far-end stopband attenuation especially when the signal needed to be filtered has a great concentration of spectral energy. In a sub-band coding, the filter is intended to separate out various frequency bands for independent processing. When it is applied on speech, the far-end rejection of the energy in the stopband needs to be as higher as possible to make leakage of the energy from one band to another as lower as possible. Therefore, the designed filter should have special specifications which should provide better far-end stopband attenuation (amplitude of last ripple in stopband). Finding a digital filter that has a higher performance far-end stopband attenuation than Kaiser Window is very valuable when the FIR filter constructed by the use of Kaiser Window far-end stopband attenuation becomes better than the one constructed by the well-known adjustable windows, for instance, the special cases of Ultra-spherical windows, Dolph-Chebyshev and Saramaki.

In this paper, the design of digital non-recursive finite impulse response (FIR) filter by using Exponential window is proposed. Also, the construction of non-recursive digital FIR filter has been presented through applying Exponential window. After applying the Exponential window, it is found that the far-end stopband attenuation becomes better than the filter constructed by Kaiser window, and that is one of the advantages of filter building by using Exponential window over Kaiser window. The proposed scheme is simulated by MATLAB. All the simulation results show a good agreement with the proposed theory.

Keywords: Digital FIR Filter, Side-lobe Roll-off Ratio, Far-end Stopband Attenuation, Window Technique, Exponential Window

INTRODUCTION

A more comprehensive view of the truncation and smoothing operations is in terms of window functions (or windows for short).Windows are normally compared and classified into different types according to their spectral characteristics. Window functions have been widely used in various digital signal processing (DSP) applications such as signal analysis, signal estimation, digital filter design and speech processing [1][2].

Various windows have been proposed to achieve the desired solutions [3][4][2][5]. Cosine hyperbolic function is one of them [6]. The idea of this window is based on the Kaiser window, but it has an advantage since there is no expanding in the power series in the time domain representation. This window gives a better ripple ratio for wider main lobe width and larger side lobe roll-off ratio along with the ultra-spherical comparison. When its function is merged with the Hamming window, it produces a better performance in terms of the ripple ratio, better than a same margin of a Kaiser and Hamming windows. Another method to design ultra-spherical window functions in order to reach prescribed spectral characteristics can be found in [4]. This method is made of combining various techniques basically to measure the ultra-spherical window, independent parameters which are ripple ratio and main-lobe width or null-to-null width along with a user-defined side-lobe pattern can also be reached. A simple comparison has been made between the ultra-spherical and Kaiser Windows and the result of this comparison showed that there is a difference in the performance which depends on the required specifications [14].

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It is well known that the Kaiser window is a flexible one which is used in applications such as digital filter design and spectrum analysis [2][6]. The advantage of using the Kaiser window is that it accomplishes a good approximation to the discrete pro late spheroidal functions whose main-lobe has a maximum concentration of energy. There are two main independent parameters in the Kaiser window: the first one is the window length (N) and the second one is the shape parameter alpha (α). For different applications, it is possible to control the main-lobe width, ripple ratio and side-lobe roll off ratio by changing these two parameters.

There are many useful adjustable windows for instance Saramaki [5] and Dolph - Chebyshev [3]. In fact, they are special form of Ultra-spherical window [7]. However, the side-lobe roll off characteristics of the Kaiser window is better than the last mentioned two windows. In some applications, it could be quite reasonable to obtain a window that could provide higher side-lobe roll off characteristics than what Kaiser Window provides.

It has been noted that the window based on exponential function offers a higher side-lobe roll off ratio compared to the Kaiser window [8] [15]. In this paper, the idea of exponential window has been explored for designing the digital Non recursive finite impulse response (FIR) filters. It is shown that the FIR filter designed with the help of exponential window provides better far-end stop band attenuation against filters designed by well-known windows in literature.

METHODOLOGY

Fir Filter Design Methods

There are five steps in the process of designing a digital filter:

i. Specifying the type of filter. For example, low pass filter the preferred amplitude and/or phase responses and the acceptable tolerances, the sampling frequency, and the length of words in the input data.

ii. Determining the coefficients of a transfer function, $H(z)$ that satisfies the specifications given in (i). There are several factors that influence the choice of the method of coefficient calculation. The critical requirements in step (i) are the most important of these factors.

iii. Converting the transfer function obtained in (ii) into a suitable filter network or structure, which is known as realization.

iv. Analysing the effects of finite word length. Here, the effect of quantizing the filter coefficients and the input data as well as the effect of carrying out the filtering operation are analysed by using fixed word lengths on the filter performance.

v. Producing the software code and/or hardware and performing the actual filtering.

These five interrelated steps are summarized in Fig. (1).

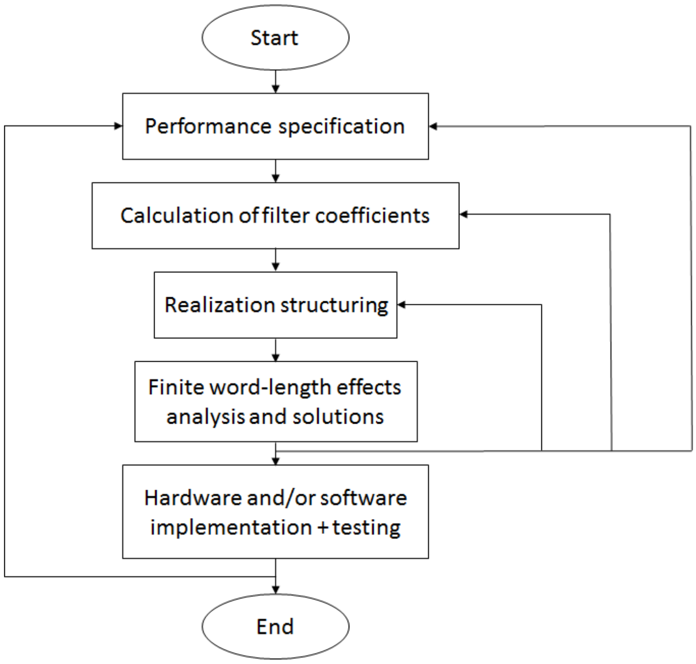

Figure1.*Summary of Design Stages for Digital Filters*

FIR Filter Design Using Exponential Window

Spectral Characteristics of Windows

A window function w(nT) having a length of N is a time-domain function which is nonzero for $n \leq |(N-1)/2|$ and zero for other values of n. The frequency spectrum of w (nT) can be defined as

$$W(e^{j\omega t}) = e^{-j\theta(\omega)} W_0(e^{j\omega t}) \tag{1}$$

Where$W_0(e^{j\omega t})$ stands for the amplitude function. The amplitude and phase spectrums of a window function are defined as A(ω) = $|W_0(e^{j\omega t})|$and θ(ω), respectively. The amplitude spectrum in the normalized form is given by$|W_0(e^{j\omega t})|/W_0\,(e^0)$.

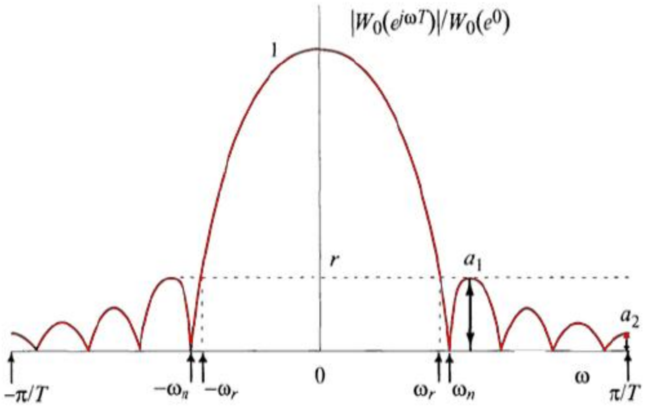

Figure2. *Amplitude Spectrum and Some Common Spectral Characteristics of a Typical Normalized Window*

Typical normalized amplitude spectrum of a window together with some spectral characteristics is shown in Fig. 2. One of the most important variables of the window is the ripple ratio (see the Fig. 2) which is defined as in the following equation:

$$r = \frac{maximum\ sidelobe\ amplitude}{main\ lobe\ amplitude} \tag{2}$$

Since the ripple ratio is a small quantity less than unity, it is more suitable to consider its larger amounts by using the reciprocal of r in decibels (dB) as:

$$R = 20\ log_{10} \frac{1}{r} \tag{3}$$

Where R represents the minimum side-lobe attenuation (or equivalently minimum stopband attenuation) with respect to the main lobe. In addition, the second parameter that describes the side-lobe pattern of a window is the side-lobe roll-off ratio whose definition is given by:

$$S = \frac{a_1}{a_2} \tag{4}$$

Wherea_1 represents the amplitude of the nearest side lobe, a_2 represents the amplitude of the furthest side lobe (see Fig. 2). The side-lobe roll-off ratio, S, can be obtained from its dB domain as follows

$$S = 10^{S/20} \tag{5}$$

In order to have a logical meaning of the side-lobe roll-off ratio, the side-lobe pattern envelope should be increased or decreased monotonically. Also, the side-lobe roll-off ratio gives an explanation of the energy distribution over the side lobes that could be considered important, when prior knowledge of the location of an interfering signal is known. More about the usefulness of this spectral characteristic can be found in [12].

Exponential Window

The plot of functions exp(x) and Io(x) (zero order Bessel function of first kind used for Kaiser Window) are shown in Fig. 3. It is clear from this figure that they exhibit the same shape characteristics which is exponential in nature. For this reason, a new window which is called Exponential window is defined as:

$$w_{ex}(k) = \frac{exp\left(\alpha_{ex}\sqrt{1-(\frac{2n}{N-1})^2}\right)}{exp(\alpha_{ex})} \quad , \quad |n| \le \frac{N-1}{2} \tag{6}$$

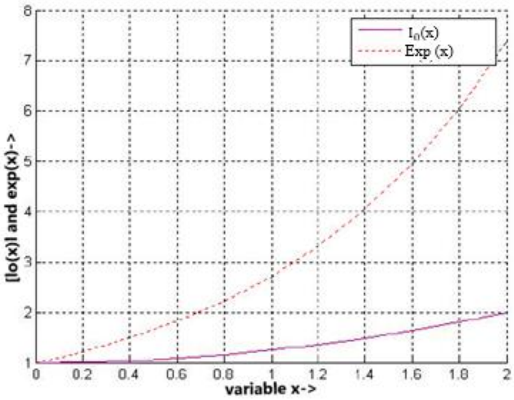

Figure3. *The Functions exp(x) and Io(x) Characteristics which have Similar Shape*

The normalized spectrum of the Exponential window in dB can be written as:

$$W_N(e^{j\omega T}) = 20\, log_{10}(\frac{|A(\omega)|}{|A(\omega)|_{max}}) \tag{7}$$

The magnitude response of the Exponential window obtained with different values of α_{ex} when the value of filter length is constant (N = 51) is shown in Figure 4. It should be noted that α_{ex}=0 corresponds to the rectangular window. It is obvious from Fig. 4 that the main-lobe width increases and ripple ratio decreases if α_{ex} is increased.

FIR Filter Design Using Exponential Window

In designing FIR filters, the most straightforward approach is the Fourier series technique. This technique requires a very small calculation in comparison to other optimization methods. The idea of using a window in Fourier series technique is to truncate and smooth the in finite duration impulse response of the ideal prototype filter. The realizable non-causal FIR filter with a window function, w (nT), has the following impulse response:

$$h_{nc}(nT) = w(nT)h_{id}(nT) \tag{8}$$

Where h_{id}(nT) represents the infinite duration impulse response of the ideal filter. The infinite duration impulse response of a low pass filter (LPF) with a cut off frequency (ω_c) and a sampling frequency (ω_s) can be obtained from [1].

$$h_{id}(nT) = \begin{cases} \frac{\omega_c T}{\pi} & ; n = 0 \\ \frac{\sin \omega_c nT}{n\pi} & ; \omega_c \le |\omega| \le \omega_s/2 \end{cases} \tag{9}$$

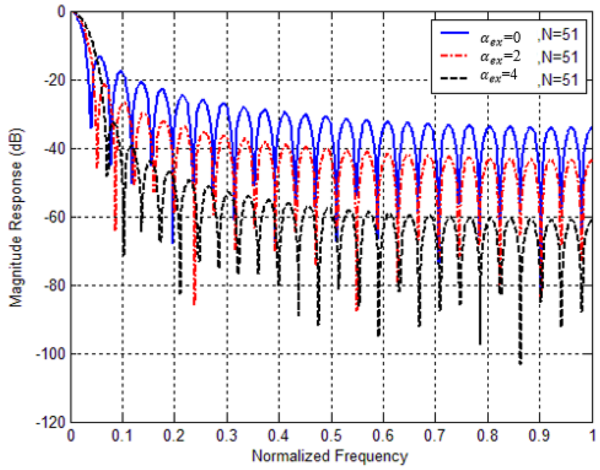

Figure41. *Proposed Window Spectrum in dB for αex= 0, 2, and 4 and N=51*

It is possible to obtain a causal filter by delaying the non-causal impulse response,$h_{nc}(nT)$, by period $(N-1)/2$. The expression of this causal filter is given by:

$$h(nT) = h_{nc}\left[\left(n-\frac{N-1}{2}\right)T\right] ;0 \le n \le N-1 \tag{10}$$

It is well known that the filters which are designed by the window method have approximately equal ripples in the pass band and stop band regions [6].

Fig. 5 shows the magnitude response of digital FIR filter designed by exponential window. The parameter (α_{ex}) effect on the filter characteristic can be clearly noticed. Also, it is clear from Fig. 5 that when $\alpha_{ex}$is increased, the minimum stopband attenuation (A_s) becomes better but the transition width becomes worse.

Filter Design Using Exponential Window Function

In order to find an appropriate window satisfying desired filter specifications, it is needed to find the relationship between the window parameters and the filter parameters. The relationship between Exponential window parameter, $\alpha_{ex}$and the minimum stopband attenuation (A_s) has been shown in Fig. 6 when N=127. It is obvious from Fig. 6 that the value of minimum stop band attenuation increases when the value of the window parameter becomes larger. The first design equation which can be obtained by applying the quadratic polynomial curve fitting method can be written as:

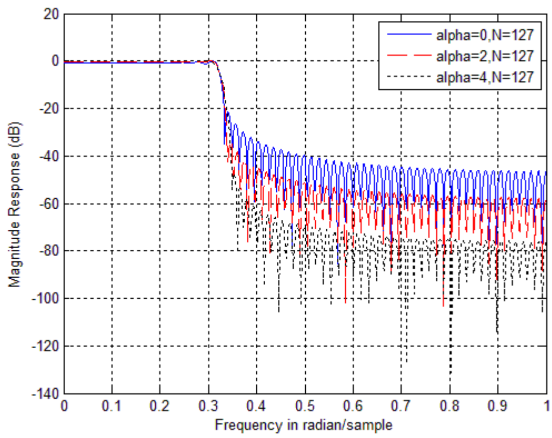

Figure5. *Amplitude Spectrums of the Filters Designed by the Exponential Window for Various Alpha with N=127*

$$\alpha_{ex} = -0.0004275\, A_s^{\,2} + 0.1808\, A_s - 3.516 \quad ; A_s \leq 20.77 \tag{11}$$

Secondly, in filter design equation, in order to find the minimum length of the filter, the relation between the normalized width, D, and the minimum stopband attenuation (A_s) should be obtained [7]. Also, the equation of the normalized width parameter can be written as:

$$D = \frac{\Delta\omega(N-1)}{\omega_s} \tag{12}$$

Where$\Delta\omega$ denotes the transition bandwidth.

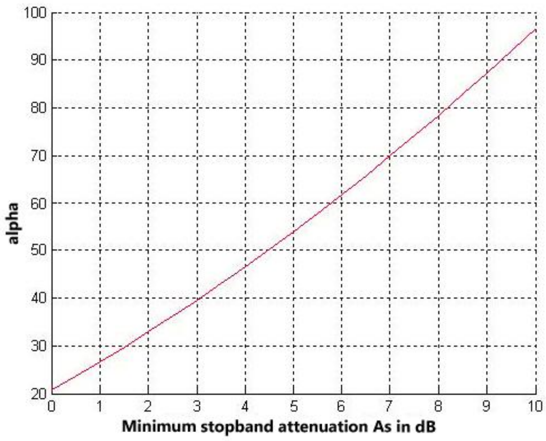

Figure6. *The Relation between Alpha and the Minimum Stop band Attenuation for Exponential Window with N=127*

Fig. 7 shows the relation between the stop band attenuation (A_s) and the normalized width (D). A comparison is also shown in Fig. 7 between the filters designed by Kaiser and Exponential windows. It can be seen from Fig. 7 that the filters designed by Kaiser Window exhibit better minimum stopband attenuation characteristic than that of designed by exponential window. An approximate equation for the normalized width (D) can be determined by using quadratic curve fitting method as follows

$$D = 5.188 \times 10^{-5} A_s^{\,2} + 0.06617\, A_s - 0.1518 \quad ; \quad A_s \leq 20.77 \tag{13}$$

The minimum odd integer filter length needed to satisfy A_s and $\Delta\omega$ can be obtained, with the help of equations (12) and (13),

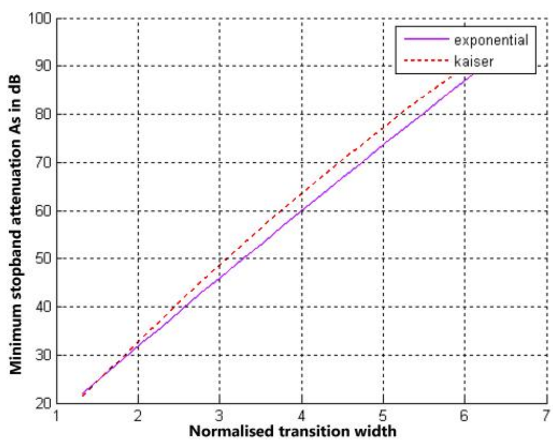

Figure7. *The Difference in the Minimum Stopband Attenuation with N=127 between the Filters Designed by Exponential and Kaiser Windows*

As a consequence, an exponential window can be designed by using equations (11), (12) and (13). It is worth to note that this exponential window is expected to satisfy the desired filter characteristic presented in terms of $\Delta\omega$and A_s.

In order to do another comparison with Kaiser Window, the far end stop band attenuation is also considered as a figure of merit. Fig. 8 shows the comparison of far end stop band attenuation in FIR filter which is designed by exponential and Kaiser Windows. It is obvious from Fig. 8 that when the transition width is increased, the filters designed by exponential window provide better far end suppression than that of designed by Kaiser Window.

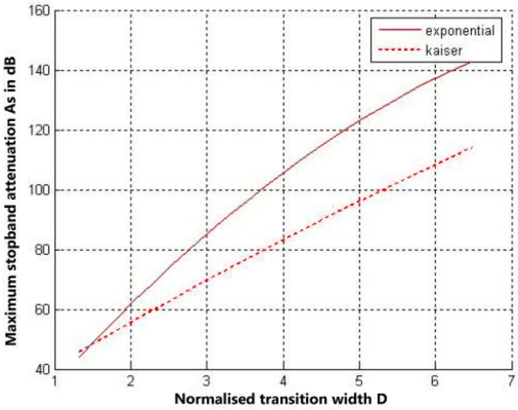

Figure8. *The Difference between the Designed Filters once when Exponential and Kaiser Windows, the term of Comparison is the Maximum Stop band Attenuation with N=127*

Computer Simulations

the simulation results carried out for designing a low pass FIR filter by using Kaiser, Cosh, and Exponential windows satisfying the following specifications are discussed:

Sampling frequency Fs=5 kHz,

passband edge frequency=100 Hz,

stopband edge frequency=150 Hz,

passband attenuation=10 dB,

stopband attenuation=60 dB,

FIR Filter Design by Kaiser Window

The simulated frequency response of the FIR filter which is designed by the Kaiser window shows in Fig. 9.

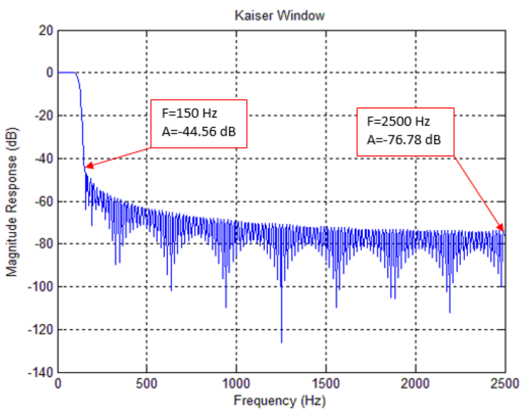

Figure9. *Frequency Response of FIR Filter Obtained by Kaiser Window, where F=frequency and A=magnitude*

To achieve a 50 Hz transition band (from 100 to 150 Hz) and at least 50 dB of attenuation (from 10 to 60 dB) in the design, the value of α_k was taken as 3.9524 and the number of coefficients (filter length) was taken as N =259. The far end stopband attenuation (FSA) is measured as FSA=-76.78dB and the minimum stopband attenuation (MSA) was measured as MSA=-44.56dB.

FIR Filter Design by Cosh Window

The frequency response of the FIR filter which is designed by the Cosh window shows in Fig. 10. To achieve a 50 Hz transition band (from 100 to 150 Hz) and at least 50 dB of attenuation (from 10 to 60 dB) in the design, the value of α_c was taken as 3.7111 and the number of coefficients was set to N =295. The far end stopband attenuation is measured as FSA=-81.27dB and the minimum stopband attenuation was measured as MSA=-48.79dB.

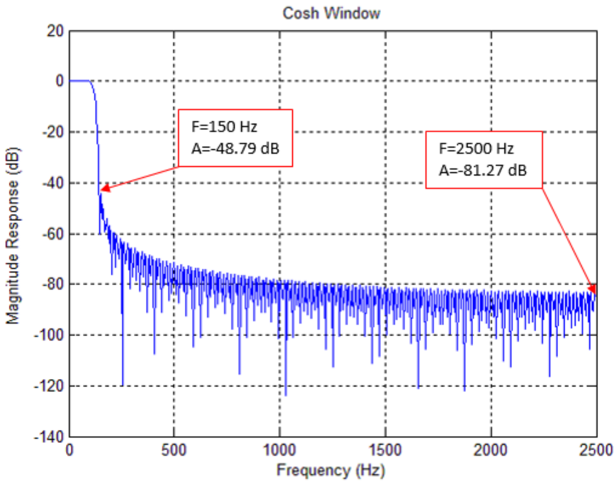

Figure10. *Frequency Response of FIR Filter Obtained by Cosh Window, where F=frequency and A=magnitude*

FIR Filter Design by Exponential Window

The frequency response of the FIR filter which is designed by the Exponential window shows in Fig. 11. To achieve a 50 Hz transition band (from 100 to 150 Hz) and at least 50 dB of attenuation (from 10 to 60 dB) in the design, the value of α_{ex} was taken as 3.7254 and the number of coefficients was set to N =293. The far end stopband attenuation is measured as FSA=-82dB and the minimum stop band attenuation was measured as MSA=-47.5dB.

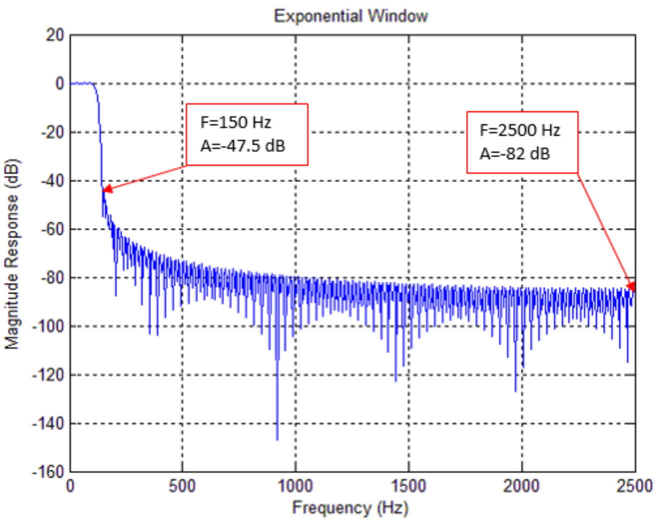

Figure11. *Frequency Response of FIR Filter Obtained by Exponential Window, where F=frequency and A=magnitude*

Table1. *Comparison of FIR Filters Designed by Kaiser, Cosh and Exponential Windows*

Parameters	Kaiser	Cosh	Exponential
ω_c	0.7695	0.7695	0.7695
D	2.5659	2.9329	2.9165
Alpha (α)	3.9524	3.7111	3.7254
N	259	295	293
FSA (in dB)	-76.78	-81.27	-82
MSA (in dB)	-44.56	-48.79	-47.5

The parameters of Kaiser, Exponential and Cosh windows together with the FSA and MSA values obtained from the simulation study are recorded in Table 1. The normalized cutoff frequency (ω_c) is computed by using the following equation

$$\omega_c = 2 \times \pi \times f_c \tag{13}$$

Wheref_c is a cutoff frequency. Normalized transition width (D) is computed by using the following equation

$$D = \frac{\Delta\omega(N-1)}{\omega_s} \tag{14}$$

Where$\Delta\omega$ is a transition bandwidth, (N) is a filter length and ω_s is sampling frequency. From table 1, it is obvious that the Exponential window provides maximum FSA than that of obtained by Kaiser and Coshwindows. However, its performance in the minimum stopband is not the best. Here, the Kaiser window provides maximum MSA than that of obtained by Exponential and Coshwindows[13].

Performance Comparison of the Filters

Performances of the FIR filters designed by the Exponential, Kaiser and Cosh window functions were tested by applying a sinusoidal input, x(n), with a frequency of 50 Hz and inspecting the output produced by the filter, the input signal and the output signals obtained from the filter designed by Kaiser, Cosh and Exponential windows in shows Fig. 12. Since the frequency of the input signal is 50Hz which is within the passband of the filter whose magnitude response is equal to one, the amplitude of the output signal remains unchanged for all window functions. However, there exists a phase shift between input and output due to the characteristics of the filter.

Again, performances of the same FIR filters were tested by applying a sinusoidal input, x(n), with a frequency of 150 Hz and inspecting the output produced by each filter.

Fig.13 shows the input signal and the output signals obtained from each filter. The frequency of the input signal is 150Hz which is the cut-off frequency of the filter. The amplitude of the output signal obtained from the filter designed by Kaiser Window is approximately6×10^{-3} which agrees well with the result obtained from

$$10^{(-44.56/20)} = 5.915 \times 10^{-3} \tag{15}$$

Where 44.56 is the MSA value in dB at 150Hz (see Fig. 9).

Similarly, the amplitude of the output signal obtained from the filter designed by Cosh window is approximately3.6×10^{-3} which agrees well with the result obtained from

$$10^{(-48.79/20)} = 3.634 \times 10^{-3} \tag{16}$$

where 48.79 is the MSA value in dB at 150Hz (see Fig. 10).

Finally, the amplitude of the output signal obtained from the filter designed by Exponential window is approximately4.4×10^{-3} which agrees well with the result obtained from

$$10^{(-47.5/20)} = 4.216 \times 10^{-3} \quad (17)$$

Where 47.5 is the MSA value in dB at 150Hz (see Fig. 11).

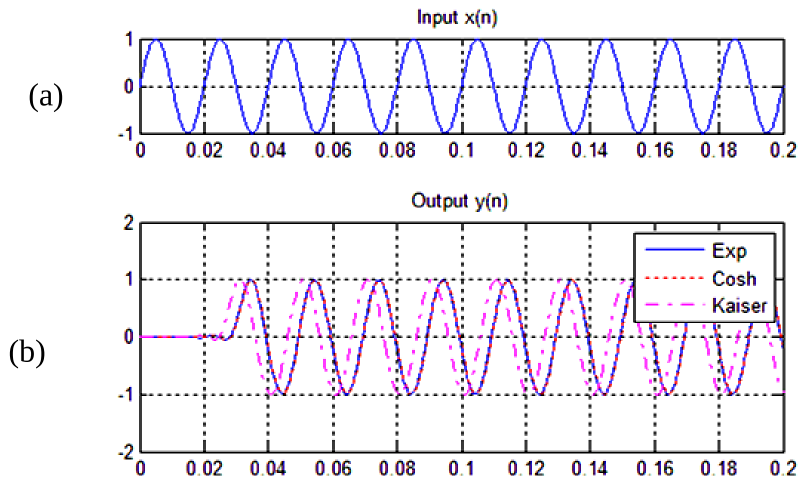

Figure12.(a) *Sinusoidal Input,* **(b)** *Outputs of the Filter Obtained by Kaiser, Cosh and Exponential windows*

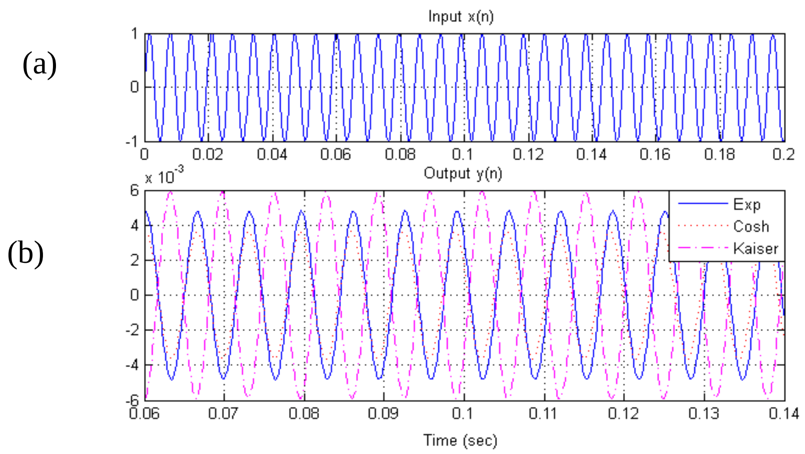

Figure13. (a) *Sinusoidal Input,* **(b)** *Outputs of the Filter Obtained by Kaiser, Cosh and Exponential windows*

The far end stop band attenuation performance of the filters can also be tested by selecting the frequency to be in the stop band (i.e. beyond the cutoff). It is clear from table2 that the amplitude of the output signal obtained from the filter designed by the Exponential window is expected to be the minimum compared with the outputs of other filters. This shows that the performance of the Exponential window is the best in the far end stop band.

CONCLUSIONS

The application of the Exponential window in the Non recursive digital FIR filter design has been introduced in this paper. It has been observed that the Exponential window exhibits worse minimum stop band attenuation than that of obtained by Kaiser Window. However, it offers better far end attenuation than the filter which is designed by Kaiser and Cosh windows. Comparisons are based on the normalized transition width, filter length, design parameter, far end stop band attenuation and minimum stop band attenuation. The better far end stop band attenuation can be considered as a significant improvement which can be used in sub-band coding and speech applications.

The minimum and far end stop band attenuation levels regarding each filter have been verified by a simulation study. The simulation results showed a good agreement with the theoretical results.

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