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Delay-Range-Dependent Stability Analysis for Continuous Linear System with Interval Delay

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ABSTRACT

This paper studies the problem of delay-range-dependent stability for continuous-time system with interval delay. Based on the dividing of the delay and reciprocally convex combination technique, some new delay-dependent stability conditions are derived by constructing a novel Lyapunov functional. These criteria are expressed as a set of linear matrix inequalities (LMIs), which can be checked using the numerically efficient Matlab LMI Control Toolbox. Finally, one numerical example is given to demonstrate the effectiveness of the proposed methods.

Keywords: Delay-dependent stability, reciprocally convex combination technique, Lyapunov functional, Interval delay.

INTRODUCTION

As is well known that stability is a central issue in dynamical system and control theory. However, the existence of time delay in dynamical system may induce instability in them. The stability problem for time-delayed systems has received considerable attention in recent years [1-13]. Especially, the stability analysis should have been focusing on effective reduction of the conservation of the stability conditions. In real system, the delay is assumed to be an interval time-varying delay, i.e. $d_1 \le d_1(t) \le d_2$ and most of the literatures always assume that $d_1 = 0$, but d_1 should not be restricted to be 0. So, delay-dependent stability for interval time-varying delay was investigated by many scholars [4-9].

Recently, various approaches have been proposed by many researchers for system with interval time-varying delay. It is well known that the model transformation technique and the bounding technology are introduced into the stability analysis, although they may lead to some conservation. In order to reduce the conservatism, the free-weighting matrix method was used in [7] to study the delay-range-dependent stability, but the free weighting matrix method makes stability criteria complicated. In [8], a less conservative delay-range-dependent stability result has been obtained and subsequently its improvement can be found in [9] by defining the new Lyapunov functional. However, the criteria proposed in [9] are based on the usage of slack variables and integral inequality technique which inevitably increases the computational burden. In [10], by dividing the delay interval into multiple segments, a new Lyapunov-Krasovskii functional is constructed with different weighting matrices corresponding to different segments. The delay-dividing method is useful for reducing conservatism of the analysis result. Recently, a less conservative stability result of time-delayed systems reported in [11] by using a reciprocally convex combination technique to estimate the derivative of the Lyapunov functional. Then, this motivates the present research to develop a novel method for the concerned systems by constructing a novel Lyapunov-Krasovskii functional via delay dividing technique and reciprocally convex combination technique.

In this paper, the problem of delay-range-dependent stability for continuous-time systems with time varying delays is investigated. A novel Lyapunov functional is constructed based on the delay-dividing method and reciprocally convex combination technique, some new delay-dependent stability conditions are derived. These criteria are expressed as linear matrix inequality, which can be solved by using standard numerical software. Finally, numerical example is given to demonstrate the

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effectiveness of the proposed method. In this paper, by using a new method based on the non smooth analysis, we obtain an improved sufficient condition for the GAS of the equilibrium point without demanding the boundedness and differentiability of activation functions. One example is provided to show the effectiveness and the benefits of the proposed method.

Notation: Throughout this paper, T stands for matrix transposition. R^n is the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $n\times m$ dimensional matrices. I denotes the identity matrix of appropriate dimensions. $P>0$ means that P is positive definite. $P\geq 0$ means that P is positive semi-definite. $*$ represents the elements below the main diagonal of a symmetric matrix.

PROBLEM STATEMENT

Consider the following continuous system with interval delay:

$$\dot{x}(t)=Ax(t)+Bx\left(t-d(t)\right), \tag{1}$$

where $x(t)\in R^n$ is the state vector, $A,B\in R^{n\times n}$ are constant matrices with appropriate dimensions. $d(t)$ is a time-varying delay, and it is assumed to satisfy

$$d_1\leq d(t)\leq d_2<\infty, \tag{2}$$

$$0\leq \dot{d}\left(t\right)\leq \nu\leq +\infty, \tag{3}$$

where d_1,d_2 and ν are constants.

The following lemmas are introduced, which will be used in the proof of the main results.

Lemma1[13]. For any constant matrix $0<R=R^T\in R^{n\times n}$, scalar $\gamma>0$, vector function $\omega:[0,\gamma]\to R^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^{\gamma}\omega(s)ds\right)^T R\left(\int_0^{\gamma}\omega(s)ds\right)\leq \gamma\int_0^{\gamma}\omega^T(s)R\omega(s)ds.$$

Lemma2[11]. For $k_i(t)\in[0,1],\sum_{i=1}^{N}k_i(t)=1$, and vectors η_i which satisfy $\eta_i=0$ with $k_i(t)=0$, matrices $R_i>0$, there exist matrix $S_{ij}(i=1,2,\cdots,N-1;\ j=i+1,\cdots,N)$, satisfies

$$\begin{pmatrix} R_i & S_{ij}\\ S_{ij}^T & R_j\end{pmatrix}\geq 0,$$

such that the following inequality holds

$$\sum_{i=1}^{N}\frac{1}{k_i(t)}\eta_i^T R_i\eta_i\geq \begin{pmatrix}\eta_1\\ \vdots\\ \eta_n\end{pmatrix}^T\begin{pmatrix} R_1 & \cdots & S_{1,N}\\ * & \ddots & \vdots\\ * & * & R_N\end{pmatrix}\begin{pmatrix}\eta_1\\ \vdots\\ \eta_n\end{pmatrix}.$$

STABILITY ANALYSIS

In this section, we firstly derive the following delay-dependent stable criterion by using the delay-dividing method and the reciprocally convex method.

Theorem1. For given scalars $d_i\geq 0$ and $\nu\geq 0(i=1,2)$, system (1) satisfying conditions (2) and (3) is asymptotically stable if there exist positive definite matrices $P>0,Q_i>0,$ $T_j>0\ \ (i=1,2;\ j=1,2,3),\ Z=\begin{pmatrix} Z_{11} & Z_{12}\\ Z_{12}^T & Z_{22}\end{pmatrix}>0,\ R=\begin{pmatrix} R_{11} & R_{12}\\ R_{12}^T & R_{22}\end{pmatrix}>0,$ and positive semi-definite matrices

$$T=\begin{pmatrix} T_2 & T_{12} & T_{13}\\ T_{12}^T & T_2 & T_{23}\\ T_{13}^T & T_{23}^T & T_2\end{pmatrix}\geq 0$$ such that the following symmetric linear matrix inequality holds:

$$\Theta = \begin{pmatrix} \Theta_1 & \Gamma^T \Lambda \\ \Lambda^T \Gamma & -\Lambda \end{pmatrix} < 0, \quad (4)$$

where

$$\Gamma = [A, 0, 0, B, 0, 0, 0], \quad \Lambda = d_1^2 T_1 + (d_2 - d_1)^2 T_2 + d_2^2 T_3,$$

$$\Theta_1 = \begin{pmatrix} \Theta_{11} & Z_{12} + 2T_1 & 0 & PB & 0 & R_{12} + 2T_3 & 0 \\ * & Z_{22} - Z_{11} - 4T_1 & -Z_{12} + 2T_1 & 0 & 0 & 0 & 0 \\ * & * & -Z_{22} - 2T_1 - T_2 & T_2 - T_{12} & T_{12} - T_{13} & 0 & T_{13} \\ * & * & * & \Theta_{44} & \Theta_{45} & 0 & -T_{13} + T_{23} \\ * & * & * & * & \Theta_{55} & 0 & -T_{23} + T_2 \\ * & * & * & * & * & \Theta_{66} & -R_{12} + 2T_3 \\ * & * & * & * & * & * & \Theta_{77} \end{pmatrix}$$

with

$$\Theta_{11} = PA + A^T P + Q_1 + Q_2 + Z_{11} + R_{11} - 2T_1 - 2T_3,$$

$$\Theta_{44} = -(1-\nu)Q_1 - 2T_2 + T_{12}^T + T_{12}, \quad \Theta_{45} = -T_{12} + T_{13} + T_2 - T_{23},$$

$$\Theta_{55} = -(1-\nu)Q_2 - 2T_2 + T_{23}^T + T_{23}, \quad \Theta_{66} = R_{22} - R_{11} - 4T_3,$$

$$\Theta_{77} = -R_{22} - 2T_3 - T_2.$$

Proof. Choose a Lyapunov functional candidate for the system (1) to be

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (5)$$

where

$$V_1(t) = x^T(t) P x(t), \quad (6)$$

$$V_2(t) = \int_{t-d(t)}^{t} x^T(s) Q_1 x(s) ds + \int_{t-d_2-d(t)}^{t} x^T(s) Q_2 x(s) ds, \quad (7)$$

$$V_3(t) = \int_{t-\frac{d_1}{2}}^{t} \begin{pmatrix} x(s) \\ x(s-\frac{d_1}{2}) \end{pmatrix}^T Z \begin{pmatrix} x(s) \\ x(s-\frac{d_1}{2}) \end{pmatrix} ds + \int_{t-\frac{d_2}{2}}^{t} \begin{pmatrix} x(s) \\ x(s-\frac{d_2}{2}) \end{pmatrix}^T R \begin{pmatrix} x(s) \\ x(s-\frac{d_2}{2}) \end{pmatrix} ds, \quad (8)$$

$$V_4(t) = \int_{-d_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) d_1 T_1 \dot{x}(s) ds d\theta + \int_{-d_2}^{-d_1} \int_{t+\theta}^{t} \dot{x}^T(s)(d_2 - d_1) T_2 \dot{x}(s) ds d\theta$$

$$+ \int_{-d_2}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) d_2 T_3 \dot{x}(s) ds d\theta, \quad (9)$$

where $P > 0, Q_i > 0,\ T_j > 0\ (i = 1,2; j = 1,2,3),\ Z > 0,\ R > 0,$ are to be determined.

Next, taking the derivative of $V(t)$ with respect to t along the solution of system (1) yields

$$\dot{V}_1(t) = 2x^T(t) P \dot{x}(t) = x^T(t)(PA + A^T P)x(t) + 2x^T(t) PBx[t - d(t)], \quad (10)$$

$$\dot{V}_2(t) \le x^T(t)(Q_1 + Q_2)x(t) - (1-\nu)x^T(t - d(t)) Q_1 x[t - d(t)]$$

$$-(1-\nu)x^T(t - d_2 - d(t)) Q_2 x[t - d_2 - d(t)], \quad (11)$$

$$\dot{V}_3(t) \le x^T(t)(Z_{11} + R_{11})x(t) + 2x^T(t) Z_{12} x\left(t - \frac{d_1}{2}\right) + x^T\left(t - \frac{d_1}{2}\right)(Z_{22} - Z_{11}) x\left(t - \frac{d_1}{2}\right)$$

$$-2x^T\left(t-\frac{d_1}{2}\right)Z_{12}x(t-d_1)-x^T(t-d_1)Z_{22}x(t-d_1)+2x^T(t)R_{12}x\left(t-\frac{d_2}{2}\right)$$

$$+x^T\left(t-\frac{d_2}{2}\right)(R_{22}-R_{11})x\left(t-\frac{d_2}{2}\right)-2x^T\left(t-\frac{d_2}{2}\right)R_{12}x(t-d_2)-x^T(t-d_2)Z_{22}x(t-d_2)\ , \qquad (12)$$

$$\dot{V}_4(t)\le \dot{x}^T(t)(d_1^2T_1+(d_2-d_1)^2T_2+d_2^2T_3)\dot{x}(t)-\int_{t-d_1}^{t}\dot{x}^T(s)d_1T_1\dot{x}(s)ds$$

$$-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds-\int_{t-d_2}^{t}\dot{x}^T(s)d_2T_3\dot{x}(s)ds\ . \qquad (13)$$

By lemma 1 and the delay-dividing approach, for (13), we can obtain

$$-\int_{t-d_1}^{t}\dot{x}^T(s)d_1T_1\dot{x}(s)ds=-2\int_{t-\frac{d_1}{2}}^{t}\dot{x}^T(s)\frac{d_1}{2}T_1\dot{x}(s)ds-2\int_{t-d_1}^{t-\frac{d_1}{2}}\dot{x}^T(s)\frac{d_1}{2}T_1\dot{x}(s)ds$$

$$\le -2\left[\left(\int_{t-\frac{d_1}{2}}^{t}\dot{x}(s)ds\right)^T T_1\left(\int_{t-\frac{d_1}{2}}^{t}\dot{x}(s)ds\right)+\left(\int_{t-d_1}^{t-\frac{d_1}{2}}\dot{x}(s)ds\right)^T T_1\left(\int_{t-d_1}^{t-\frac{d_1}{2}}\dot{x}(s)ds\right)\right]$$

$$=2\left[x^T(t)(-T_1)x(t)+2x^T(t)T_1x\left(t-\frac{d_1}{2}\right)+x^T\left(t-\frac{d_1}{2}\right)(-2T_1)x\left(t-\frac{d_1}{2}\right)\right.$$

$$\left.+2x^T\left(t-\frac{d_1}{2}\right)T_1x(t-d_1)+x^T(t-d_1)(-T_1)x(t-d_1)\right], \qquad (14)$$

$$-\int_{t-d_2}^{t}\dot{x}^T(s)d_2T_3\dot{x}(s)ds=-2\int_{t-\frac{d_2}{2}}^{t}\dot{x}^T(s)\frac{d_2}{2}T_3\dot{x}(s)ds-2\int_{t-d_2}^{t-\frac{d_2}{2}}\dot{x}^T(s)\frac{d_2}{2}T_3\dot{x}(s)ds$$

$$\le -2\left[\left(\int_{t-\frac{d_2}{2}}^{t}\dot{x}(s)ds\right)^T T_3\left(\int_{t-\frac{d_2}{2}}^{t}\dot{x}(s)ds\right)+\left(\int_{t-d_2}^{t-\frac{d_2}{2}}\dot{x}(s)ds\right)^T T_3\left(\int_{t-d_2}^{t-\frac{d_2}{2}}\dot{x}(s)ds\right)\right]$$

$$=2\left[x^T(t)(-T_3)x(t)+2x^T(t)T_3x\left(t-\frac{d_2}{2}\right)+x^T\left(t-\frac{d_2}{2}\right)(-2T_3)x\left(t-\frac{d_2}{2}\right)\right.$$

$$\left.+2x^T\left(t-\frac{d_2}{2}\right)T_3x(t-d_2)+x^T(t-d_2)(-T_3)x(t-d_2)\right]. \qquad (15)$$

By the reciprocally convex approach, for the other terms of (13), we can get

$$-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds$$

$$=-\int_{t-d(t)}^{t-d_1}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds-\int_{t-d_2-d(t)}^{t-d(t)}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds$$

$$-\int_{t-d(t)}^{t-d_2-d(t)}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds\ . \qquad (16)$$

It is noted that

$$\frac{d(t)-d_1}{d_2-d_1}+\frac{d_2+d(t)-d_1}{d_2-d_1}+\frac{d_2-d_2-d(t)}{d_2-d_1}=1\ .$$

From lemma 2, it is easy to see that (16) can be written the following inequality, respectively:

$$-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds$$

$$\le -\begin{pmatrix} x(t-d_1)-x(t-d(t)) \\ x(t-d(t))-x(t-d_2-d(t)) \\ x(t-d_2-d(t))-x(t-d_2) \end{pmatrix}^T \begin{pmatrix} T_2 & T_{12} & T_{13} \\ T_{12}^T & T_2 & T_{23} \\ T_{13}^T & T_{23}^T & T_2 \end{pmatrix} \begin{pmatrix} x(t-d_1)-x(t-d(t)) \\ x(t-d(t))-x(t-d_2-d(t)) \\ x(t-d_2-d(t))-x(t-d_2) \end{pmatrix}. \quad (17)$$

Then combing equations (10)-(17), we derive

$$\dot{V}(t)=\xi^T(t)\ \Theta_1+\Gamma^T\Lambda\Gamma\ \xi(t), \quad (18)$$

where

$$\xi(t)=\left(x^T(t),x^T\left(t-\frac{d_1}{2}\right),x^T(t-d_1),x^T(t-d(t)),x^T(t-d_2-d(t)),x^T\left(t-\frac{d_2}{2}\right),x^T(t-d_2)\right)^T. \quad (19)$$

By virtue of the Schur complement Lemma, the inequality (18) is equivalent to (4) which results in $\dot{V}(t)<0$ from (18). Therefore, according to Hale [14], if there exist symmetric positive definite matrices $P>0, Q_i>0,\ T_j>0\ \ (i=1,2; j=1,2,3),\ Z>0,\ R>0$ such that the LMI (4) is satisfied, then system (1) with time-varying delays $d(t)$ satisfying (2) and (3) is asymptotically stable. This completes the proof.

Remark1. Theorem 1 gives a delay-dependent and rate-dependent stability criterion for system (1) by employing the delay-dividing method and the reciprocally convex method as in [15]. In this paper, by dividing the delay $[0,d],\ [0,d_1],\ [0,d_2]$ into two segments, the information of $\frac{d}{2},\frac{d_1}{2},\frac{d_2}{2}$ is fully considered and a new Lyapunov-Krasovskii functional is constructed. Compared with those in [7-9, 12], the method in this paper is more useful. In addition, the reciprocally convex technique [11] was utilized in $V_4(t)$ which transfer the term $-\int_{t-d_2}^{t-d_1}\dot{x}^T(s)(d_2-d_1)T_2\dot{x}(s)ds$ to (26). Then, the information of the bounds of time delays are fullly explored which may lead less conservative results.

Remark2. For unknown $d(t)$, the corresponding result is given by Theorems 2 with $Q_1=Q_2=0$.

Theorem2. For given scalars $d_i\ge 0\ \ (i=1,2)$, system (1) satisfying conditions (2) is asymptotically stable if there exist positive definite matrices $P>0,\ T_1>0,\ T_2>0,$

$T_3>0,\ Z=\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{pmatrix}>0,\ R=\begin{pmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{pmatrix}>0,$ and positive semi-definite matrices

$T=\begin{pmatrix} T_2 & T_{12} & T_{13} \\ T_{12}^T & T_2 & T_{23} \\ T_{13}^T & T_{23}^T & T_2 \end{pmatrix}>0$ such that the following symmetric linear matrix inequality holds:

$$\hat{\Theta}=\begin{pmatrix} \hat{\Theta}_1 & \Gamma^T\Lambda \\ \Lambda^T\Gamma & -\Lambda \end{pmatrix}<0, \quad (20)$$

where

$$\Gamma=\ A,0,0,B,0,0,0\ ,\ \ \Lambda=d_1^2T_1+(d_2-d_1)^2T_2+d_2^2T_3,$$

$$\hat{\Theta}_1=\begin{pmatrix} \hat{\Theta}_{11} & Z_{12}+2T_1 & 0 & PB & 0 & R_{12}+2T_3 & 0 \\ * & Z_{22}-Z_{11}-4T_1 & -Z_{12}+2T & 0 & 0 & 0 & 0 \\ * & * & -Z_{22}-2T_1-T_2 & T_2-T_{12} & T_{12}-T_{13} & 0 & T_{13} \\ * & * & * & \hat{\Theta}_{44} & \Theta_{45} & 0 & -T_{13}+T_{23} \\ * & * & * & * & \hat{\Theta}_{55} & 0 & -T_{23}+T_2 \\ * & * & * & * & * & \Theta_{66} & -R_{12}+2T_3 \\ * & * & * & * & * & * & \Theta_{77} \end{pmatrix}$$

with

$\widehat{\Theta}_{11} = PA + A^T P + Z_{11} + R_{11} - 2T_1 - 2T_3,$

$\widehat{\Theta}_{44} = -2T_2 + T_{12}^T + T_{12},\ \widehat{\Theta}_{55} = -2T_2 + T_{23}^T + T_{23},$

where Θ_{45}, Θ_{66} and Θ_{77} are defined the same as Theorem 1.

ILLUSTRATIVE EXAMPLE

In this section, we use one example and compare our results with the previous ones to show the effectiveness of ours.

Example. Consider the following system as in [9] with:

$$A = \begin{pmatrix} 0 & 1 \\ -10 & -2 \end{pmatrix},\ B = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix},$$

$d_1 \le d(t) \le d_2.$

CaseI. For $\nu = 0.3$, we calculate the upper bounds d_2 to guarantee asymptotic stability of the above system for different values of d_1. It can be seen from Table 1 that the upper bounds d_2 increases as d_1 increases. In addition, It is easy to see that Theorem 1 gives much better results than those obtained by [7-9].

Table1. *Calculated delay bounds for different cases*

Method	d_1	2	3	4	5
He [7]	d_2	2.4091	3.3342	4.2799	5.2393
Shao [8]	d_2	2.4798	3.3893	4.3250	5.2773
Lin [9]	d_2	2.58	3.47	4.39	5.33
Theorem 1	d_2	2.6026	3.5005	4.4281	5.3738

Case II. In the case of unknown ν, the upper bounds d_2 obtained from Theorem 2 is listed in Table 2 for different values of d_1. It is clear that the obtained results in our paper are significantly better than those in [7-9, 12].

Table2. *Calculated delay bounds for different cases*

Method	d_1	0.3	0.5	0.8	1	2
Jiang [12]	d_2	0.91	1.07	1.33	1.5	2.39
He [7]	d_2	0.9431	1.0991	1.3476	1.5187	2.4000
Shao [8]	d_2	1.0715	1.2191	1.4539	1.6169	2.4798
Lin [9]	d_2	1.24	1.38	1.60	1.75	2.58
Theorem 2	d_2	1.24	1.38	1.60	1.7606	2.6026

CONCLUSION

This paper has discussed the asymptotical stability problem for continuous-time system with interval delay. Based on a novel Lyapunov functional method and linear matrix inequality technology, delay-dependent stability conditions are derived by using a delay-dividing approach and reciprocally convex combination technique. One numerical example is given to demonstrate the effectiveness of the proposed methods.

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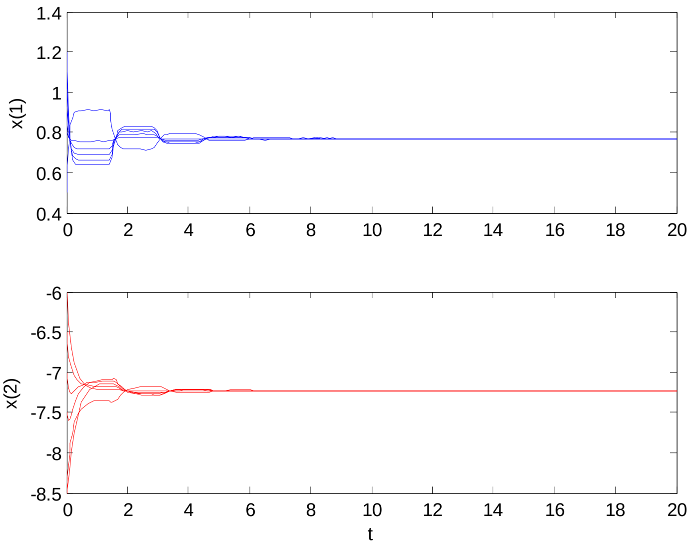

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