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New Global Asymptotic Stability Condition for Delayed Neural Networks with a Constant Delay

Chunxia Li , Fang Qiu

Department of Mathematics, Binzhou University, Shandong 256603, P. R. China

ABSTRACT

This paper investigates the global asymptotical stability problems for delayed neural networks. By introducing some triple integral terms in the constructing of the Lyapunov-Krasovskii functional, combined with the inequality analysis, a new asymptotical stability condition in terms of linear matrix inequalities are proposed, Finally, numerical example is given to demonstrate the effectiveness of the proposed methods by the Matlab LMI Control Toolbox.

Keywords: Neural networks, Triple integral terms, Lyapunov functional, global asymptotical stability.

INTRODUCTION

In recent decades, the research of the stability of delayed neural networks (DNNs) has become an attractive issue [1-14]. Various methods have been proposed to obtain criteria ensuring the global asymptotical stability of the equilibrium point, such as the free-weighting matrix method, M-matrix, the linear matrix inequality approach, the delay-dividing approach. In [4], based on the linear matrix inequality approach, several stability results with constant or time-varying delays are obtained. In [11]， the authors used the free-weighting matrix method to study robust stability of neutral system with mixed time-varying delays and nonlinear perturbations, combining with novel Lyapunov functional, some stability conditions with less conservatism were established, but the free weighting matrix method makes stability criteria complicated.

Recently, some triple terms were introduced in the constructing of the Lyapunov functional, which improves the traditional Lyapunov functional and play a key role in the improvement of less conservative results [12-14]. In [13], by constructing Lyapunov functional that contains some triple terms and combining with integral inequality, stability results that were better than [12] have been obtained. Based on this method, the authors in [14] studied a class of neural network with a constant delay, and the obtained conditions were less conservative. Thus, it is necessary to further study the stability analysis of DNNs by using this novel Lyapunov functional which motivates for this paper.

In this paper, the global asymptotic stability of the DNNs is concerned. By constructing the novel Lyapunov functional that contains some triple integral terms, we develop a new stability criterion, the effectiveness can be illustrated with numerical example.

Notation: Throughout this paper, T stands for matrix transposition. R^n Is the n-dimensional Euclidean space. $P>0$ Means that P is positive definite. The notation $*$ denotes the symmetric term in a symmetric matrix.

PROBLEM STATEMENT

Consider the following neural network with constant delay:

$$\dot{w}(t) = -Cw(t) + Ag(w(t)) + Bg(w(t-h)) + J\ , \tag{1}$$

Where $w(t) = (w_1(t), w_2(t), \cdots, w_n(t))^T \in R^n$ is the state vector; $C = diag\{c_1, c_2, \cdots, c_n\}$ is a

Address for correspondence:

lichunxia2138@126.com

Diagonal matrix; $A=(a_{ij})_{n\times n}$ is the feedback matrix and $B=(b_{ij})_{n\times n}$ is the delayed feedback matrix; $J=(J_1,J_2,\cdots,J_n)^T$ is a constant input vector; $g(w(t))=(g_1(w_1(t)),g_2(w_2(t)),\cdots,g_n(w_n(t)))^T$

Represents the activation function; h is the time delay.

Throughout this paper, we assume that each of the activation functions $g_i(i=1,2,\cdots,n)$ satisfies the following hypothesis A:

(A_1) g_i is a bounded function for any $i=1,2,\cdots,n$;

(A_2) $0<\dfrac{g_i(u)-g_i(v)}{u-v}<k_i$, $u\neq v\in R$, $i=1,2,\cdots,n$.

Next, through the transformation $x(t)=w(t)-w^*$, the equilibrium point $w^*=(w_1^*,w_2^*,\cdots,w_n^*)$ of system (1) is shifted to the origin, and then system (1) can be transformed to the following system:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-h)) \tag{2}$$

Where $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in R^n$, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\cdots,f_n(x_n(t)))^T\in R^n$ with

$$f_i(x_i(t))=g_i(x_i(t)+w_i^*)-g_i(w_i^*),\ \ i=1,2,\cdots,n.$$

Noted that each $g_i(i=1,2,\cdots,n)$ satisfies the hypothesis A, hence, each function

$f_i(t)(i=1,2,\cdots,n)$ Satisfies:

$$0<\frac{f_i(x_i)}{x_i}<k_i,\quad f_i(0)=0,\ \ \forall x_i\neq 0,\ i=1,2,\cdots,n, \tag{3}$$

Which is equivalent to?

$$f_i(x_i)(f_i(x_i)-k_ix_i)\leq 0,\ \ f_i(0)=0,\ \forall x_i\neq 0,\ i=1,2,\cdots,n. \tag{4}$$

First, we give the following lemma, which is useful in the proof of the result.

Lemma 1[15]. For any constant matrix $W>0$, scalars $a<b$ and vector function $\omega(s):[a,b]\to R^n$ such that the following integrations are well defined, then

$$(b-a)\int_a^b\omega^T(s)W\omega(s)ds\geq\int_a^b\omega^T(s)dsW\int_a^b\omega(s)ds;$$

$$\frac{(b-a)^2}{2}\int_a^b\int_\theta^b\omega^T(s)W\omega(s)dsd\theta\geq\int_a^b\int_\theta^b\omega^T(s)dsd\theta W\int_a^b\int_\theta^b\omega(s)dsd\theta;$$

$$\frac{(b-a)^2}{2}\int_a^b\int_a^\theta\omega^T(s)W\omega(s)dsd\theta\geq\int_a^b\int_a^\theta\omega^T(s)dsd\theta W\int_a^b\int_a^\theta\omega(s)dsd\theta.$$

STABILITY ANALYSIS

In this section, by introducing novel Lyapunov functional combined with the triple integral terms, we present a new asymptotical stability criterion for system (1).

Theorem 1. For a given scalar $h\geq 0$, system (1) with fixed matrices A,B and C is asymptotically stable if there exist diagonal matrices $S=diag(s_i)_{n\times n}\geq 0$, $\Lambda_j\geq 0(j=1,2)$ and positive definite matrices $P=(P_{ij})_{2\times 2}>0,Q_i>0,R_i>0,Z_i>0(i=1,2)$, such that

$$\begin{pmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & -P_{22}^{\ T}+hZ_2 & 0 & K\Lambda_2 \\ * & * & \Omega_{33} & P_{12}A & P_{12}B \\ * & * & * & \Omega_{44} & A^TYB+SB \\ * & * & * & * & \Omega_{55} \end{pmatrix} < 0 \tag{5}$$

where

$$\Omega_{11} = -P_{11}C + P_{12} - C^T P_{11}^{\ T} + P_{12}^{\ T} + C^T YC - R_1 - h^2 Z_1 + Q_2 + h^2 R_2,$$

$$\Omega_{12} = -P_{12} + R_1, \quad \Omega_{13} = -C^T P_{12}^{\ T} + P_{22}^{\ T} + hZ_1,$$

$$\Omega_{14} = P_{11}A - C^T YA - C^T S + K\Lambda_1, \quad \Omega_{15} = P_{11}B - C^T YB,$$

$$\Omega_{22} = -R_1 - h^2 Z_2 - Q_2, \quad \Omega_{33} = -Z_1 - Z_2 - R_2,$$

$$\Omega_{44} = A^T YA + A^T S + SA + Q_1 - 2\Lambda_1, \quad \Omega_{55} = B^T YB - Q_1 - 2\Lambda_2 .$$

with $Y = h^2 R_1 + \dfrac{h^4}{4} Z_1 + \dfrac{h^4}{4} Z_{2.}$

Proof. Choose a Lyapunov functional candidate for the system (1) to be

$$V(t) = \sum_{i=1}^{5} V_i(t),$$

where

$$V_1(t) = \xi^T(t) P \xi(t),$$

$$V_2(t) = 2\sum_{i=1}^{n} s_i \int_0^{x_i(t)} f_i^T(\alpha)d\alpha + \int_{t-h}^{t} f^T(x(\alpha))Q_1 f(x(\alpha))d\alpha,$$

$$V_3(t) = \int_{t-h}^{t} x^T(\alpha)Q_2 x(\alpha)d\alpha,$$

$$V_4(t) = h\int_{-h}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha d\beta + h\int_{-h}^{0}\int_{t+\beta}^{t} x^T(\alpha)R_2 x(\alpha)d\alpha d\beta,$$

$$V_5(t) = \frac{h^2}{2}\int_{-h}^{0}\int_{\gamma}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)Z_1\dot{x}(\alpha)d\alpha d\beta d\gamma + \frac{h^2}{2}\int_{-h}^{0}\int_{-h}^{\gamma}\int_{t+\beta}^{t} \dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha d\beta d\gamma .$$

Where $\xi(t) = \{x(t), \int_{t-h}^{t} x(\alpha)d\alpha\}^T, P = (P_{ij})_{2\times 2} > 0, Q_i > 0, R_i > 0, Z_i > 0 (i = 1,2),$ and diagonal matrix $S \geq 0$ are to be determined.

From (5), there exists appropriately dimensioned diagonal matrices $\Lambda_j \geq 0 (j = 1,2)$ and $K = diag\{k_1, k_2, \cdots, k_n\}$, such that

$$-2f^T(x(t))\Lambda_1[f(x(t))-Kx(t)]\geq 0, \tag{6}$$

$$-2f^T(x(t-h))\Lambda_2[f(x(t-h))-Kx(t-h)]\geq 0, \tag{7}$$

Calculate the time derivative of $V(t)$ along the trajectories of system (1), then

$$\dot{V}_1(t)=2\xi^T(t)P\dot{\xi}(t)=2\begin{pmatrix} x(t) \\ \int_{t-h}^{t} x(\alpha)d\alpha \end{pmatrix}^T \begin{pmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{pmatrix}\begin{pmatrix} \dot{x}(t) \\ x(t)-x(t-h) \end{pmatrix}, \tag{8}$$

$$\dot{V}_2(t)=2f^T(x(t))S\dot{x}(t)+f^T(x(t))Q_1f(x(t))-f^T(x(t-h))Q_1f(x(t-h))$$

$$=2f^T(x(t))S(-Cx(t)+Af(x(t))+Bf(x(t-h))\ +\ f^T(x(t))Q_1f(x(t))-f^T(x(t-h))Q_1f(x(t-h)), \tag{9}$$

$$\dot{V}_3(t)=x^T(t)Q_2x(t)-x^T(t-h))Q_2x(t-h), \tag{10}$$

$$\dot{V}_4(t)=h^2\dot{x}^T(t)R_1\dot{x}(t)-h\int_{t-h}^{t}\dot{x}^T(\alpha)d\alpha R_1\dot{x}(\alpha)d\alpha\ +h^2x^T(t)R_2x(t)-h\int_{t-h}^{t}x^T(\alpha)d\alpha R_2x(\alpha)d\alpha \tag{11}$$

$$\dot{V}_5(t)=\frac{h^4}{4}\dot{x}^T(t)z_1\dot{x}(t)-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z_1\dot{x}(\alpha)d\alpha d\beta$$

$$+\frac{h^4}{4}\dot{x}^T(t)z_2\dot{x}(t)-\frac{h^2}{2}\int_{-h}^{0}\int_{t-h}^{t+\beta}\dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha d\beta. \tag{12}$$

Using Lemma 1, the following inequalities hold

$$-h\int_{t-h}^{t}x^T(\alpha)R_2x(\alpha)d\alpha\leq-\int_{t-h}^{t}x^T(\alpha)d\alpha R_2\int_{t-h}^{t}x(\alpha)d\alpha, \tag{13}$$

$$-h\int_{t-h}^{t}\dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha\leq-\int_{t-h}^{t}\dot{x}^T(\alpha)d\alpha R_1\int_{t-h}^{t}\dot{x}(\alpha)d\alpha$$

$$=(x^T(t)-x^T(t-h))(-R_1)(x(t)-x(t-h)) \tag{14}$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\beta}^{t}\dot{x}^T(\alpha)Z_1\dot{x}(\alpha)d\alpha d\beta\leq-\int_{-h}^{0}\int_{t+\beta}^{t}\dot{x}^T(\alpha)d\alpha d\beta Z_1\int_{-h}^{0}\int_{t+\beta}^{t}\dot{x}(\alpha)d\alpha d\beta$$

$$=(hx^T(t)-\int_{t-h}^{t}x^T(\alpha)d\alpha)(-Z_1)(hx(t)-\int_{t-h}^{t}x(\alpha)d\alpha) \tag{15}$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t-h}^{t+\beta}\dot{x}^T(\alpha)Z_2\dot{x}(\alpha)d\alpha d\beta\leq-\int_{-h}^{0}\int_{t-h}^{t+\beta}\dot{x}^T(\alpha)d\alpha d\beta Z_2\int_{-h}^{0}\int_{t-h}^{t+\beta}\dot{x}(\alpha)d\alpha d\beta$$

$$=(\int_{t-h}^{t}x^T(\alpha)d\alpha-hx^T(t-h))(-Z_2)(\int_{t-h}^{t}x(\alpha)d\alpha-hx(t-h)) \tag{16}$$

From the above conditions (6)-(7), (13)-(16), we have

$$\dot{V}(t)\leq\sum_{i=1}^{5}\dot{V}_i(t)-2f^T(x(t))\Lambda_1[f(x(t)-Kx(t)]$$

$$-2f^T(x(t-h))\Lambda_2[f(x(t-h)-Kx(t-h)]\ \leq\chi^T(t)\Omega\chi(t).$$

Where

$$\chi(t) = col\{x(t), x(t-h), \int_{t-h}^{t} x(\alpha)d\alpha, f(x(t)), f(x(t-h))\}.$$

Thus, under the condition that Ω is a negative definite matrix, $\dot{V}(t)$ is negative for any possible state. Then the origin of (2) or equivalently the equilibrium point of (1) is globally asymptotically stable. This completes the proof.

ILLUSTRATIVE EXAMPLE

In this section, we use one example to show the effectiveness of ours.

Example. Consider the following delayed neural networks with:

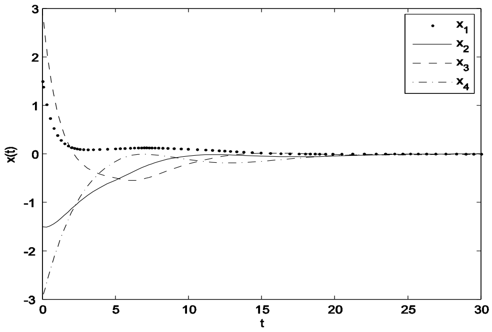
$$C = \{1.2769, 0.6231, 0.9230, 0.4480\},$$

$$A = \begin{pmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{pmatrix},$$

$$K = \{0.1137, 0.1279, 0.7994, 0.2386\}.$$

By using the Mat lab LMI Control Toolbox, we obtain the maximal value that guarantee the neural systems stable is $h = 4.6573$.



Global asymptotic stability of the system for Example

CONCLUSION

This paper has discussed the stability criteria for neural network with constant time delay. We introduce some triple integral terms in the Lyapunov-Krasovskii functional, a new stability criterion in the terms of linear matrix inequalities is derived to guarantee the neural systems stable. A numerical example is given to show the effectiveness of the proposed methods.

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