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The Homomorphism and Anti-Homomorphism of N-Generated Fuzzy Groups and Its Level Subgroups

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ABSTRACT

In this paper, we introduce some properties of level subgroups of n-generated fuzzy subgroups of a group with respect to the homomorphism and anti-homomorphism.

Keywords: Fuzzy set, multi-fuzzy set, fuzzy subgroup, multi-fuzzy subgroup, anti-fuzzy subgroup, multi-anti fuzzy subgroup, $n-$ generated fuzzy subset, $n-$ generated fuzzy subgroups, $n-$ generated fuzzy level subsets, n-generated fuzzy level subgroups, homomorphism and anti-homomorphism

INTRODUCTION

After the introduction of the concept of fuzzy sets by L.A.Zadeh [1], researchers were conducted the generalizations of the notion of fuzzy sets, A. Rosenfeld [2] introduced the concept of fuzzy group and the idea of "intuitionistic fuzzy set" was first published by K.T. Atanassov [3]. S. sabu sebastian and T.V Ramakrishnan [4,5] introduced the concept of Multi-fuzzy sets and Multi-fuzzy subgroups.Also R.Muthuraj and S.Balamurugan [6] produced some results in Multi-fuzzy groups and its lower level subgroups.Choudhury.F.P. and Chakraborty.A.B. And Khare.S.S. [7] Defined a fuzzy subgroup and fuzzy homomorphism.In this chapter we introduce some properties of $n-$ generated fuzzy subgroups of a group with homomorphism and anti-homomorphism

PRELIMINARIES

Definition

Let X be a non-empty set. A fuzzy set A on X is a mapping $A: X \to 0,1$ and is defined as $A = \ x \in X / \ x, \mu(x)$

Definition

Let X and Y be any two sets. Let $f: X \to Y$ be a function. If μ is a fuzzy set on X then the image of μ under f is a fuzzy set on Y defined by $f(\mu)\ y = v(y) = \underset{x \in f^{-1}(y)}{Sup} \mu(x), \ \forall \ y \in Y$ is called image of μ under f

Definition

Let X and Y be any two sets. Let $f: X \to Y$ b a function. If S is a fuzzy set on Y then the preimage of S under f is a fuzzy set on X & is defined by $f^{-1}(S)\ (x) = S\ f(x)$.

Definition

Let A be a fuzzy subset of a set X. For $t \in [0, 1]$, $A_t = \ x \in X / A(x) \geq t$ is called a level fuzzy subset of A

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Definition

Let X be a non-empty set, J be an indexing set and $L_j : j \in J$ a family of partially ordered sets. A multi-fuzzy set A in X is a set

$$A = \left\{ \left\langle x,\ \mu_j(x)_{j \in J} \right\rangle : x \in X, \mu_j \in L_j^X, j \in J \right\}$$

Definition

Let X be a non-empty set and A be a multi-fuzzy set on X. An $n-$ generated fuzzy set on X is of the form

$$A^G = \langle x, \lambda(x) \rangle : x \in X,\ \mu_i \in L_i^X,\ i \in P, n \in N$$

Where $0 \le \lambda(x) \le 1\ \ \forall x \in X, n > 0,\ \ \lambda(x) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(x)$

Definition

Let X and Y be any two sets. Let $f : X \to Y$ be a function. If λ is a $n-$ generated fuzzy set on X then the image of λ under f is a $n-$ generated fuzzy set on Y and is defined by $f(\lambda)\ y\ = v(y) = \underset{x \in f^{-1}(y)}{Sup}\ \lambda(x),\ \forall\ y \in Y$ is called image of λ under f

If S is an $n-$ generated fuzzy set on Y then the preimage of

X is defined by $f^{-1}(S)\ (x) = S\ \ f(x)$

Definition

Let A be an n-generated fuzzy set on X. For $t \in [0,\ 1]$, $A_t^G = \ x \in X / \lambda(x) \ge t$ is called a n-generated level fuzzy subset of A

Properties

(1). $A^G \subseteq B^G \ \Leftrightarrow\ \lambda\ x\ \le \gamma(x)$

(2). $A^G = B^G \ \Leftrightarrow\ \lambda\ x\ = \gamma(x)$

(3). $A^G \cup B^G = \lambda\ x\ \cup \gamma(x) = \left[\ x, \max\left[\lambda\ x\ , \gamma(x)\right]\ ; x \in X\ \right]$

(4). $A^G \cap B^G = \lambda\ x\ \cap \gamma(x) = \left[\ x, \min\left[\lambda\ x\ , \gamma(x)\right]\ ; x \in X\ \right]$

(5). $A + B = \left[\ x,\ \lambda\ x\ + \gamma(x) - \lambda\ x\ \gamma(x)\ \ ; x \in X\ \right]$

(6). If $A^G = \ x, \lambda\ x\ \ ; x \in X$, then $A^{G\ C} = \ x, 1 - \lambda\ x\ \ ; x \in X$

Definition

Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if

$(i).\ A(xy) \ge \min\ \ A(x), A(y)$

$(ii).\ A\left(x^{-1}\right) \ge A(x) \quad \forall\ x, y \in G$

Definition:

Let G be a group. A $n-$ generated fuzzy subset λ of a group G is called a $n-$ generated fuzzy subgroup of G if

$(i).\lambda(xy) \geq \min \ \lambda(x), \lambda(y)$ $\qquad (ii).\lambda \ x^{-1} \ = \lambda(x) \quad \forall \ x, y \in G$

where $\lambda \ x \ = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(x), \ \ \lambda(y) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(y)$, & $\lambda(xy) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(xy)$

Definition

Let G be a group. A $n-$generated fuzzy subset λ of a group G is called a $n-$generated anti- fuzzy subgroup of G if

$(i).\lambda(xy) \leq \max \ \lambda(x), \lambda(y)$

$(ii).\lambda \ x^{-1} \ = \lambda(x) \quad \forall \ x, y \in G$

Definition

Let $G, \bullet$ and $G', \bullet$ be any two groups, then the function $f : G \rightarrow G'$ is called a group homomorphism if $f \ xy \ = f(x) f(y) \quad \forall x, y \in G$

Definition

Let $G, \bullet$ and $G', \bullet$ be any two groups, then thefunction $f : G \rightarrow G'$ is called a group anti- homomorphism if $f \ xy \ = f(y) f(x) \quad \forall x, y \in G$

Theorem

Let G and G' be any two groups with identity. Let $f : G \rightarrow G'$ be a homomorphism then (i). $f(1) = 1'$,where 1 and 1' are the identities of G and G' respectively.

(ii). $f(a^{-1}) = \ f(a) \ ^{-1}$

Proposition

Let A be a fuzzy subgroup of a group G.Then for $t \in [0, 1]$ suchthat $t \leq \mu(e), A_t$ is a subgroup of G

Proposition

The homomorphic image of a fuzzy subgroup of a group G is a fuzzy subgroup of a group G'

Proposition

The homomorphic pre- image of a fuzzy subgroup of a group G' is a fuzzy subgroup of a group G

Proposition

The anti- homomorphic image of a fuzzy subgroup of a group G is a fuzzy subgroup of a group G'

Proposition

The anti- homomorphic pre- image of a fuzzy subgroup of a group G' is a fuzzy subgroup of a group G

Theorem

The homomorphic image of a level subgroup of a n-generated fuzzy subgroup of a group G is a level subgroup of an n-generated fuzzy subgroup of a group G'

Proof

Let G and G' be any two groups

Let $f : G \to G'$ be a homomorphism

That is $f(xy) = f(x)f(y) \quad \forall x, y \in G$

Let $V = f(\lambda)$, where λ an n- generated fuzzy subgroup of a group G

Clearly V is an n- generated fuzzy subgroup of G'

Let $x, y \in G \Rightarrow f(x)$ and $f(y)$ in G'

Clearly λ_t is a level subgroup of λ

That is $\lambda(x) \geq t$ and $\lambda(y) \geq t$ and $\lambda(xy^{-1}) \geq t$

We have to prove that $f(\lambda_t)$ is a level subgroup of V

Now, $V(f(x)) \geq \lambda(x) \geq t \Rightarrow V(f(x)) \geq t$

$V(f(y)) \geq \lambda(y) \geq t \Rightarrow V(f(y)) \geq t$ And

$V(f(x)(f(y))^{-1}) = V(f(x)f(y^{-1}))$, As f is a hom

$= V(f(xy^{-1}))$, as f is a hom

$\geq \lambda(xy^{-1}) \geq t$

$\Rightarrow V(f(x)(f(y))^{-1}) \geq t$

$f(\lambda_t)$ Is a level subgroup of an n-generated fuzzy subgroup of? G'

Theorem

The homomorphic pre- image of a level subgroup of a n-generated fuzzy subgroup of a group G' is a level subgroup of a n-generated fuzzy subgroup of a group G

Proof

Let G and G' be any two groups

Let $f : G \to G'$ be a homomorphism

That is $f(xy) = f(x)f(y) \quad \forall x, y \in G$

Let $V = f(\lambda)$, where λ an n- generated fuzzy subgroup of a group G'

Clearly λ is an n- generated fuzzy subgroup of G

Let $f(x)$ and $f(y)$ in $G' \Rightarrow x, y \in G$

Clearly $f(\lambda_t)$ is a level subgroup of V

That is $V(f(x)) \geq t$ & $V(f(y)) \geq t$,

And $V(f(x)(f(y))^{-1}) \geq t$

We have to prove that λ_t is a level subgroup of λ

Now,

$\lambda(x) \geq V(f(x)) \geq t \Rightarrow \lambda(x) \geq t$

$\lambda\ y\ \geq V\ f(y)\ \geq t \Rightarrow \lambda\ y\ \geq t$ And

$\lambda\ xy^{-1}\ = V\ f\ xy^{-1}$

$= V\ f(x) f(y^{-1})$ As f is a homomorphism

$= V\ f(x)\ f(y)^{-1}\ \geq t$

λ_t Is a level subgroup of an n- generated fuzzy subgroup λ of a group? G'

Theorem

The anti- homomorphic image of a level subgroup of a n-generated subgroup of a group G is a level subgroup of an n-generated fuzzy subgroup of a group G'

Proof

Let G and G' be any two groups

Let $f : G \to G'$ be an anti- homomorphism

That is $f\ xy\ = f(y) f(x)\ \ \forall x, y \in G$

Let $V = f\ \lambda$, where λ an n- generated fuzzy subgroup of group G

Clearly V is an n- generated fuzzy subgroup of G'

Let $x, y \in G \Rightarrow f(x)$ and $f(y)$ in G'

Clearly λ_t is a level subgroup of λ

That is $\lambda(x) \geq t$ and $\lambda(y) \geq t$ and $\lambda(y^{-1}x) \geq t$

We have to prove that $f\ \lambda_t$ is a level subgroup of V

Now, $V\ f(x)\ \geq \lambda(x) \geq t \Rightarrow V\ f(x)\ \geq t$

$V\ f(y)\ \geq \lambda(y) \geq t \Rightarrow V\ f(y)\ \geq t$ And

$V\ f(y)^{-1} f(x)\ = V\ f(y^{-1}) f(x)$, As f is a anti- hom

$= V\ f(y^{-1}x)$ As f is a anti- hom

$\geq \lambda(y^{-1}x) \geq t$

$\Rightarrow V\ f(y)^{-1} f(x)\ \geq t$

$f\ \lambda_t$ Is a level subgroup of an n-generated fuzzy subgroup V of a group? G'

Theorem

The anti- homomorphic pre- image of a level subgroup of a n-generated fuzzy subgroup of a group G' is a level subgroup of a n-generated fuzzy subgroup of a group G

Proof

Let G and G' be any two groups

Let $f : G \to G'$ be an anti- homomorphism

That is $f\ xy\ = f(y) f(x)\ \ \forall x, y \in G$

Let $V = f(\lambda)$, where λ an n- generated fuzzy subgroup of is G'

Clearly λ is an n- generated fuzzy subgroup of G

Let $f(x)$ and $f(y)$ in $G' \Rightarrow x, y \in G$

Clearly $f(\lambda_t)$ is a level subgroup of V

That is $V(f(x)) \geq t$ & $V(f(y)) \geq t$,

And $V((f(y))^{-1} f(x)) \geq t$

We have to prove that λ_t is a level subgroup of λ

$\lambda(x) \geq V(f(x)) \geq t \Rightarrow \lambda(x) \geq t$

$\lambda(y) \geq V(f(y)) \geq t \Rightarrow \lambda(y) \geq t$ And

$\lambda(xy^{-1}) = V(f(xy^{-1}))$

$= V(f(y^{-1})f(x))$ As f is a homomorphism

$= V((f(y))^{-1} f(x)) \geq t \Rightarrow \lambda(xy^{-1}) \geq t$

λ_t Is a level subgroup of an n- generated fuzzy sub group λ of a group? G

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