

## Tracking Mobile Emitter Using TDOA and FDOA Techniques

Aous Y. Ali<sup>1</sup>, Ibrahim Beram Jasim<sup>2</sup>

<sup>1</sup>Communication and Computer Engineering Department, Cihan University, Erbil, Iraq

<sup>2</sup>Computer Engineering Department, Kitab University, Karkuk, Iraq

### ABSTRACT

Mobility devices are very important in wireless communication systems, development of accurate and reliable mobile positioning technologies. The performance of accurate location estimation is by creating methods and techniques deals with tracking of mobile emitter using a sequence of time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurement. In this paper one emitter is considered to be applied. The measurements of TDOA are defined by a region of possible emitter locations around a unique hyperbola and then the function is approximated by Gaussian Mixture. The FDOA measurements estimate of separated Kalman filters. Probability density function approximation by a Gaussian mixture and tracking results near the Cramér–Rao lower bound results in a better track state. The performance of proposed Gaussian mixture approach is evaluated using a simulation study, and compared with a bank of EKF filters and the Cramér–Rao lower bound by using Matlab. Study Time Difference of Arrival (TDOA) technique and Frequency Difference of Arrival (FDOA) technique for localizing the emitter and proposal for enhancement in existing model by applying a new module of tracking the emitter using TDOA and FDOA technique.

**Keywords:** TDOA, FDOA, Cramér–Rao and Kalman filters.

### INTRODUCTION

Localization of an emitter on the surface of the Earth (geolocation) enables important applications, both military (surveillance) and civilian (localization, law enforcement, search and rescue, etc.). In sonar and radar, it is often of interest to determine the location of an object from its emissions. A number of spatially separated sensors capture the emitted signal and the time differences of arrival (TDOA's) at the sensors are determined. Using the TDOA's, emitter location relative to the sensors can be calculated. The position fix is simplified when the sensors are arranged in a linear fashion. Many optimum processing techniques have been proposed, with different complexity and restrictions. The location system generally consists of a number of spatially well separated receivers that capture the radiated or reflected signal from the object. Due to their large coverage, localization of a close to Earth object by satellites has become popular in recent years. The geolocation systems currently in operation are VOR/DME, OMEGA, LORAN C, GPS, GLONASS, and GEOSTAR, with each of them having different coverage and accuracy. These systems are originally developed for search and rescue, and the military. However, some of them such as GPS are available for civilian applications after being intentionally degraded in accuracy for a better visibility of emitters, it is often advantageous to mount sensors on unmanned aerial vehicles (UAVs). The UAVs may use small omnidirectional antennas and measure the time of arrival of signals at the receiver. A single measurement of this type is unable to provide any emitter-location information. When two sensors receive the same signal, the time difference of arrival (TDOA) can be calculated. Knowing the TDOA between the two sensors geolocalizes the emitter to a region around the points of a hyperbola. The TDOA measurements are especially suited to the geolocation of high-bandwidth emitters, e.g., radars. With the introduction of additional sensors (additional TDOA measurements), the emitter geolocation can be estimated at the intersection of two or more hyperbolae. [1][2][3]

Various algorithms have been implemented in finding the device location accurate. A method called as time difference of arrival (TDOA) is used in which the source localization is accurately found by the intersection of the two hyper bodies generated by the emitter sources. The difference between time of arrival (TOA) and time difference of arrival (TDOA) is also shown. [3]

*\*Address for correspondence:*

Aous.younis@yahoo.com

In this paper we are employing an algorithm for implementing the TDOA process for finding the non-stationary sources accurately previously many techniques have been implemented to find the location of an emitter. In this dissertation a non-stationary emitter is considered. The localization of such an emitter is tedious and requires accurate measurement for localizing and tracking. Hence, for high bandwidth emitters techniques like TDOA and FDOA extended with GMM-ITS is used for solving this nonlinear estimation. [3][5][6].

## **METHODOLOGY**

### **Signal Parameters in Geolocation**

The parameters generally measured by the ES system for a pulsed signal include carrier radio frequency (RF), pulse amplitude (PA), pulse width (PW), time of arrival (TOA), and angle of arrival (AOA). In some systems, polarization of the input signal is measured. Furthermore, frequency-modulation- on the - pulse (FMOP) is another parameter that can be used to identify a particular emitter and also can be used to determine the chirp rate or phase coding of a pulse compression (PC) signal by definition continuous wave (CW) signals are generally identified as those signals whose pulse lengths exceed several hundred microseconds. TDOA measurements are made with respect to an internal clock on the leading edge of the pulse. Parameters measured on a single intercept are called pulse descriptor words (PDW). The PDW form a set of vectors in the parameter space. By matching vectors from multiple pulses, it is possible to isolate those signals associated with a particular emitter. This process is called deinterleaving once a signal is isolated, an additional set of signal parameters can be derived. These include:

1. The pulse repetition frequency (PRF) or its pattern (from multiple TOAs),
2. Antenna beam width from multiple PAs,
3. Antenna scans rate or type from multiple PAs,
4. Mode switching from multiple PWs and TOA, and
5. Emitter range from multiple AOAs.

### **Emitter Identification**

Emitters are identified by comparing the characteristics derived from the intercept emissions (e.g., frequency, average PRI, PRI type, scan rate, scan type,) with those from known emitters that are stored in an emitter library residing in the ES system computer. However, at times there will be more than one emitter in the library having parameter ranges that include those of the emitter being identified. In these cases, the intercepted emitter's parameters are compared with those associated with other emitters in the environment to effect the match. For example assume a threat missile is an identification candidate and one of the other emitters is a platform radar associated with a particular threat, and they both fall in the same AOA bin. [12] The tentative identification is probably correct. If none of the identification candidates for the new emitter can be correlated with any of the other emitters in the environment, then the emitter is given the identification of that particular candidate having the greatest threat potential. [14].

### **Tracking**

Generally tracking is the observing of persons or objects on the move and supplying a timely ordered sequence of respective location data to a model e.g. capable to serve for depicting the motion on a display capability. In virtual space technology, a tracking system is generally a system capable of rendering virtual space to a human observer while tracking the observer's body coordinates. For instance, in dynamic virtual auditory space simulations, a real-time head tracker provides feedback to the central processor, allowing for selection of appropriate head-related transfer functions at the estimated current position of the observer relative to the environment. [10][11]

### **Time of arrival (TOA)**

Time of Arrival (TOA or ToA), also named Time of flight (TOF), refers to the travel time of a radio signal from a single transmitter to a remote single receiver. By the relation between light speed in vacuum and the carrier frequency of a signal the time is a measure for the distance between transmitter and receiver. However, in some publications the fact is ignored, that this relation is well

defined for vacuum, but is different for all other material when radio waves pass through. Ways of synchronization as with TDOA, synchronization of the network base station with the locating reference stations is important. This synchronization can be done in different ways:

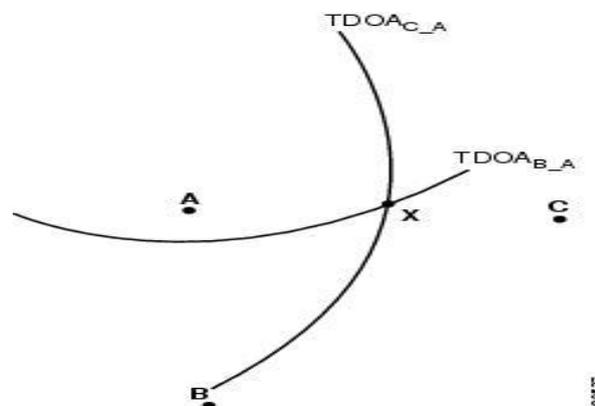
1. With exact synchronous clock on both sides. Inaccuracy in the clock synchronization translates directly to an imprecise location.
2. With two signals which have different frequencies and hence spreading speed. Distance to a lightning strike can be measured in this way (speed of light and sound velocity).
3. Via measurement to or triggering from a common reference point.
4. Without direct synchronization, but with compensation of clock phase differences

### Time Difference of Arrival

TDOA techniques are based on estimating the difference in the arrival times of the signal from the source at multiple receivers. This is usually accomplished by taking a snapshot of the signal at a synchronized time period at multiple receivers. The cross-correlation of the two versions of the signal at pairs of base stations is done and the peak of the cross correlation output gives the time difference for the signal arrival at those two base stations. A particular value of the time difference estimate defines a hyperbola between the two receivers on which the mobile may exist, assuming that the source and the receivers are coplanar. If this procedure is done again with another receiver in combination with any of the previously used receivers, another hyperbola is defined and the intersection of the two hyperbolas results in the position location estimate of the source, this method is also sometimes called a hyperbolic position location method. The below figure illustrates how the intersection of the two hyperbolas TDOAC-A and TDOAB-A is used to resolve the position of station X.

### FDOA

Frequency difference of arrival (FDOA), also frequently called differential Doppler (DD), is a technique analogous to TDOA for estimating the location of a radio emitter based on observations from other points. (It can also be used for estimating one's own position based on observations of multiple emitters). TDOA and FDOA are sometimes used together to improve location accuracy and the resulting estimates are somewhat independent. By combining TDOA and FDOA measurements, instantaneous geolocation can be performed in two dimensions. It differs from TDOA in that the FDOA observation points must be in relative motion with respect to each other and the emitter. This relative motion results in different Doppler shifts observations of the emitter at each location in general. The relative motion can be achieved by using airborne observations in aircraft, for example.



**Figure 1.** The intersection of the two hyperbolas TDOAC-A and TDOAB-A is used to resolve the position of station X.

The emitter location can then be estimated with knowledge of the observation points' location and vector velocities and the observed relative Doppler shifts between pairs of locations. A disadvantage of FDOA is that large amounts of data must be moved between observation points or to a central location to do the cross-correlation that is necessary to estimate the Doppler shift. The accuracy of the

location estimate is related to the bandwidth of the emitter's signal, the signal-to-noise ratio at each observation point, and the geometry and vector velocities of the emitter and the observation points.

### **Kalman Filtering**

The Kalman filter produces estimates of the true values of measurements and their associated calculated values by predicting a value, estimating the uncertainty of the predicted value, and computing a weighted average of the predicted value and the measured value. The most weight is given to the value with the least uncertainty. The estimates produced by the method tend to be closer to the true values than the original measurements because the weighted average has a better estimated uncertainty than either of the values that went into the weighted average.

### **Gaussian mixture model**

The most popular approach to estimate the maximum likelihood parameters of a GMM from a given data is the Expectation-Maximization (EM) algorithm. Using this approach to approximate the TDOA pdf by a GMM for each microphone pair at each time frame  $t$ , however, would be computationally expensive. Thus, we use a computationally less expensive method that provides comparable results to those obtained with the EM algorithm. Presented a Gaussian mixture model of the TDOA which couples the detection and tracking stages to enhance TDOA estimates. More specifically, our study shows that the proposed model can efficiently be used to improve the performance of acoustic source tracking algorithms, as it reduces the problem of erroneous TDOA estimates by incorporating the prior information given by the predicted pdf of the TDOA. In this work, our focus was on single source tracking problem. Future work will investigate the generalization of this approach to multiple source tracking problem.

### **GMM in this paper**

Passive measurements generally have non-Gaussian uncertainty in the observation space, i.e. they usually are nonlinear. In the measurement space, TDOA and FDOA true value uncertainties given the measurement are Gaussian, However, the transformation into the observation linear space, in this case the two-dimensional Cartesian plane, results in very non-Gaussian probability density functions (pdfs), as indicated by the uncertainty curves on Figures. Estimation using these measurements becomes non-linear information fusion, which in this work is performed using the Gaussian Measurement Mixture (GMM) algorithm. GMM filter is based on the notion that any probability density function (pdf) may be modeled by a Gaussian mixture. Estimated pdf based on non-linear (non-Gaussian) measurements is also non-Gaussian. Thus both state estimate and the observation space measurement pdfs need to be modeled by Gaussian mixtures. Each element of the Gaussian mixture is termed here a “component”. State estimate here is termed a “track”. GMM filter.

In this application both TDOA and FDOA measurements arrive simultaneously at time  $k$ , One way to use both measurements is to introduce a “dummy” time  $k+1$ , with zero seconds of physical time between time  $k$  and  $k+1$ . First the GMM estimate based on the TDOA measurement is updated at time  $k$ , and then the GMM prediction is applied between time  $k$  and  $k+1$ , and finally the FDOA measurement is applied to update the GMM state estimate at time  $k+1$ . As the time interval between samples  $k$  and  $k+1$  is zero, GMM prediction at time  $k+1$  is identical to GMM estimate at time  $k$ . Denote by  $Z_k$  the measurement received at time  $k$ .

TDOA or FDOA in this case), and by  $Z_k$  the set of all measurements received up to and including the measurement received at time  $k$ . A posteriori track pdf at time  $k-1$  (after processing the measurement  $Z_{k-1}$ ) is a Gaussian mixture, given by:

$$P(X_{k-1}|Z^{k-1}) = \sum_{c=1}^{C_k} \xi(c)N(X_{k-1}; \hat{X}_{k-1|k-1}(c), P_{k-1|k-1}(c)) \quad (1)$$

### **TDOA/FDOA measurement GMM presentation**

The same procedure is used for GMM presentation of both TDOA and FDOA measurements. In this section TDOA measurement symbols only are used. The first step involves mapping the measurements into regions in the surveillance domain. It involves drawing two parametric uncertainty curves. This procedure starts by dividing each uncertainty curve by a set of points, where both sets have the same cardinality. Then an ellipsis is inscribed within each quadrangle formed by one pair of

points on each uncertainty curve. Assume that points  $x_1$  and  $x_2$  are on one curve, and points  $x_3$  and  $x_4$  are on the other curve, and we want to define the measurement component  $g$  whose footprint is the inscribed ellipsis. The measurement component is defined by its mean  $y_k(g)$  and covariance  $R_k(g)$ .

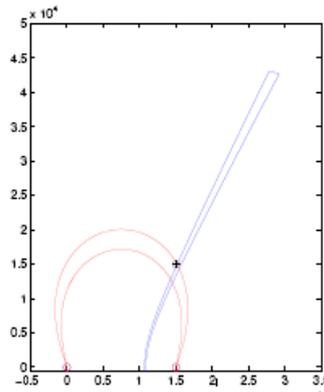


Figure 2. TDOA (blue) and FDOA (red)  $\pm\sigma$  emitter location uncertainty

The end points of one semi axis of the inscribed ellipsis are defined by

$$x_{c1} = \frac{(x_1 + x_3)}{2} \tag{2}$$

$$x_{c2} = \frac{(x_2 + x_4)}{2} \tag{3}$$

The length and the angle of one semi-axis of the ellipsis are given by

$$\Delta x_c = x_{c1} - x_{c2} \tag{4}$$

$$D_c = \frac{\|\Delta x_c\|}{2} \tag{5}$$

$$i(\alpha_c) \triangleq [\cos(\alpha_c) \sin(\alpha_c)] = \Delta x_c / \|\Delta x_c\| \tag{6}$$

The length of the other semi axis is given by

$$D_s = \frac{[-\sin(\alpha_c) \cos(\alpha_c)] \left( \frac{(X_1 - X_3)}{2} + \frac{(X_2 - X_4)}{2} \right)}{2} = \frac{i \left( \alpha_c + \frac{\pi}{2} \right)^T (X_1 + X_2 - X_3 - X_4)}{4} \tag{7}$$

Denote by  $T(\alpha) = \begin{bmatrix} i(\alpha) & i(\alpha + \frac{\pi}{2}) \end{bmatrix}$  the rotation matrix. Then the center of the inscribed ellipsis is given by

$$\hat{y}_k(g) = 0.54(X_{c1} + X_{c2})$$

Which is also the mean of the measurement component corresponding to the ellipsis the covariance matrix of the measurement component is given by.

$$R_k(g) = T(\alpha_c) \begin{bmatrix} D_c^2 & 0 \\ 0 & D_s^2 \end{bmatrix} T(\alpha_c)^T \tag{8}$$

The end result of following this procedure to transform the TDOA and FDOA measurement uncertainties from Figure 3.5 is shown on Figure 3.6, where each measurement component is represented by its ellipsis footprint. Without any prior information, the emitter position is equally probable at any point of the observation space. Therefore, the probability that the emitter is within the footprint of a measurement component is proportional to the area of the footprint.

$$\gamma(g) \propto \sqrt{|R_k(g)|} \tag{9}$$

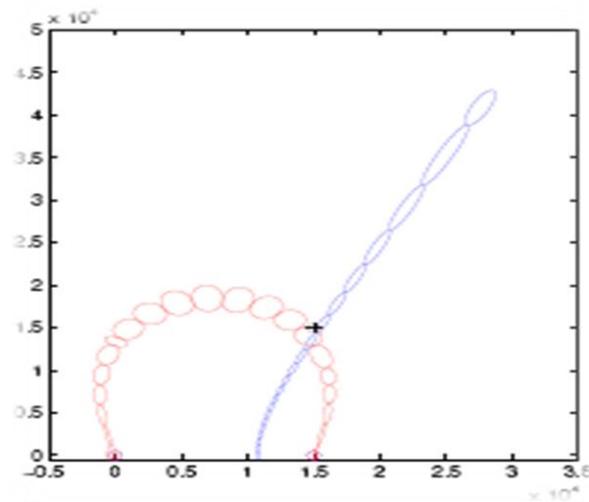


Figure3. TDOA (blue) and FDOA (red) emitter location uncertainty GMM

### Computer Simulations

The results obtained by using the proposed approach(Integration of TDOA and FDOA tracking system and by using Gaussian Mixture Model(GMM) and applying Extended Kalman Filter(EKF) are shown in figure 4.

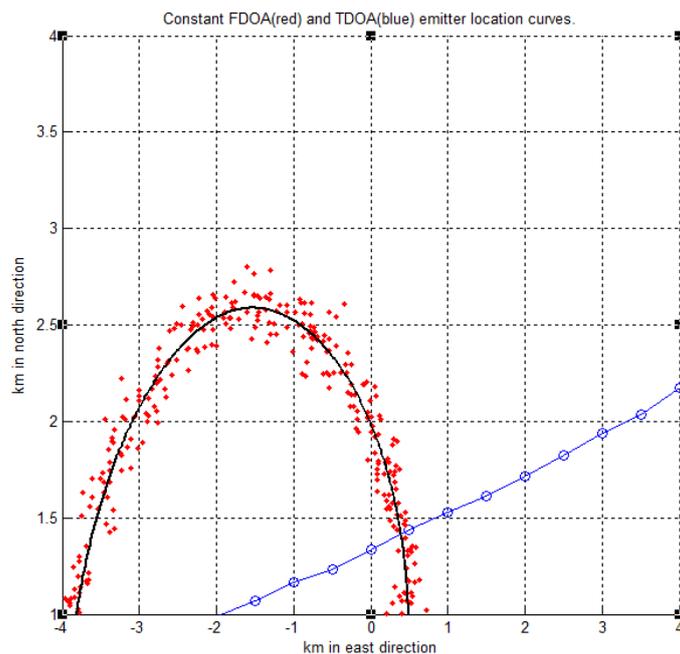


Figure4. Constant TDOA and FDOA curves

All the points on the solid line have the same distance difference to the two sensors and, therefore, the same true time difference of arrival. In all other simulation the “TDOA” results are not useful, due to large estimation errors. The “EKfb” results are significantly better. However the “TFDOA” results further significantly decrease estimation errors. Furthermore, the performance of “TFDOA” nears the theoretical optimum of the “CRLB” curve, at least in the “zoomed in” area of interest. The final “TFDOA” rms estimation errors of 3.8 and 10.5 m for the case of minimal and increased measurement errors respectively in this scenario (with the emitter more than 15 km away) are certainly a useful outcome.

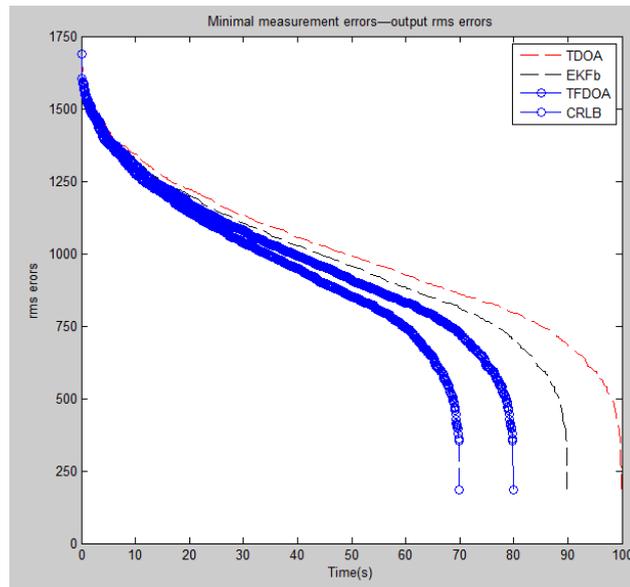


Figure 5. Minimal measurement errors – output rms errors

The position rms estimation errors and estimation bias for the case of minimal TDOA measurement errors are presented in Figure 5.

As can be seen in Figure 5, the TDOA measurements are akin to the bearings only measurements. The assumption is that the single emitter moves with uniform motion (constant velocity) and that the sensors perform maneuvers to ensure observability.

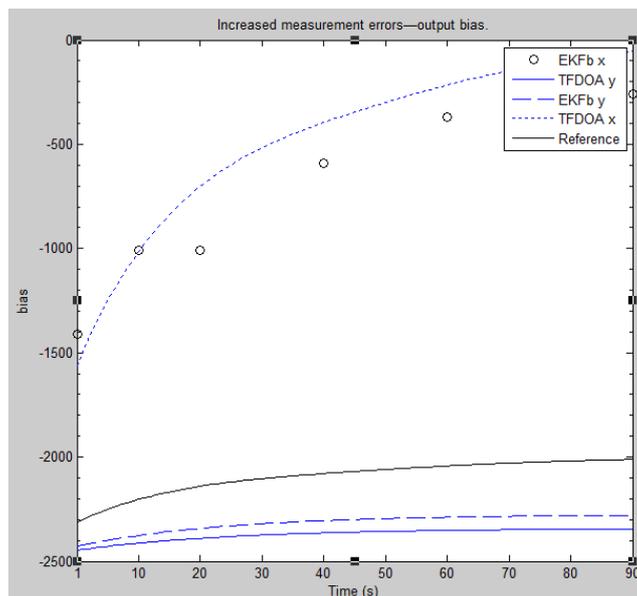


Figure 6. Minimal measurement errors – output

The position rms estimation errors and estimation bias for the case of minimal FDOA measurement errors are presented in Figure 6 in this simulation experiment consists of 1000 simulation runs, each providing 40 pairs of TDOA and FDOA measurements. Each simulation experiments consists of 40 000 filter updates. Execution times for the “EKFB” simulation experiments were 270 and 340 s for the minimal and increased measurement errors. This corresponds to 6.8 and 8.5 ms respectively per filter update. The “TFDOA” corresponding execution times were 1600 and 2300 s, which corresponds to 40 and 58 ms per filter update respectively. This fits comfortably within the real-time requirements of 2 s per filter update.

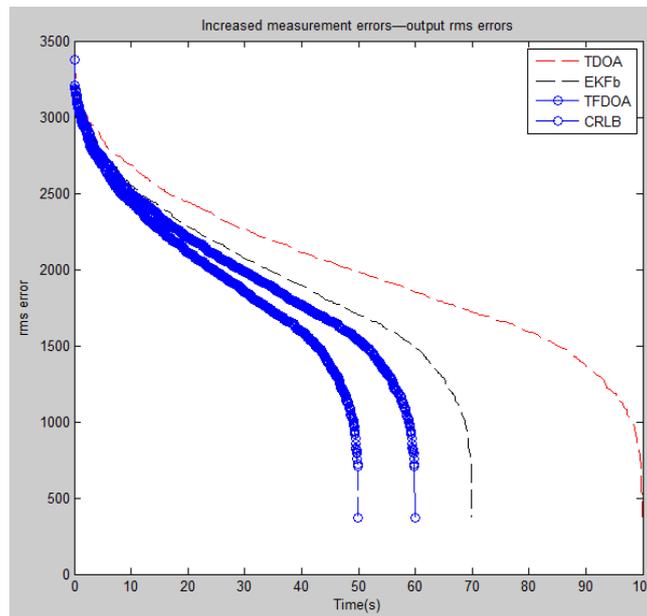


Figure 7. Increased measurement errors – output rms errors

The position rms estimation errors and estimation bias for the case of increased TDOA measurement errors are presented in Figure 7. In this scenario the “TDOA” results are not useful, due to large estimation errors. The “EKFB” results are significantly better. However, the “TFDOA” results further significantly decrease estimation errors. Furthermore, the performance of “TFDOA” nears the theoretical optimum of the “CRLB” curve, at least in the “zoomed in” area of interest. The final “TFDOA” rms estimation errors of 3.8 and 10.5 m for the case of minimal and increased measurement errors respectively in this scenario (with the emitter more than 15 km away) are certainly a useful outcome.

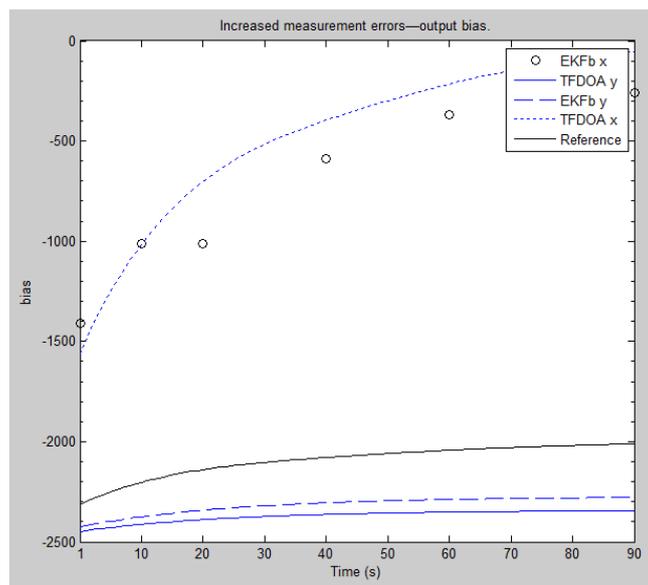


Figure 8. Increased measurement errors – output bias

The position rms estimation errors and estimation bias for the case of increased FDOA measurement errors are presented in Figure 8.

## CONCLUSION

The accuracy of the location estimate is related to the bandwidth of the emitter's signal, and TDOA and FDOA are determining the location of an object from its emissions the TDOA measurements are nonlinear, emitter position estimation using the TDOA measurement is performed by essentially linear operations, i.e., Kalman filter update.

The results of this paper presented by non-Gaussian state estimate non-Gaussian TDOA measurement by Gaussian mixtures, and also by using a dynamic Kalman filters which have small covariance. Method proposed of filtering can be accomplished in real time with only modest computational resources. The last part of this paper shown performance of the proposed algorithm, , significantly improves upon the EKF based industry standard, and is near theoretical Cramér–Rao bounds.

## REFERENCES

- [1] M. Schmidt, “A new approach to geometry of range difference location,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 8, no. 6, pp. 821–835, Nov. 1972.
- [2] K.Ho and Y.Chan, “Solution and performance analysis of geolocation by TDOA,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, no. 4, pp. 1311–1322, Oct. 1993.
- [3] Darko Mušicki, “Mobile Emitter Geolocation and Tracking Using TDOA and FDOA Measurements” *IEEE transactions on signal processing*, vol. 58, no. 3, march 2010.
- [4] G. Yao, Z. Liu, and Y. Xu, “TDOA/FDOA joint estimation in a correlated noise environment,” in *IEEE Int. Symp. Microwave, Antenna, Propagation EMC Technol. Wireless Commun.*, Beijing, China, Aug. 2005.
- [5] K. C. Ho, X. Lu, and L. Kovavisaruch, “Source localization using TDOA and FDOA measurements in the presence of receiver location errors: Analysis and solution,” *IEEE Trans. Signal Process.*, vol. 55, no. 2, pp. 684–696, Feb. 2007.
- [6] K. C. Ho and W. Xu, “An accurate algebraic solution for moving source location using TDOA and FDOA measurements,” *IEEE Trans. Signal Process.*, vol. 52, no. 9, pp. 2453–2463, Sep. 2004.
- [7] K. Becker, “Three-dimensional target motion analysis using angle and frequency measurements,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 41, no. 1, pp. 284–301, Jan. 2005.
- [8] K. Becker, “An efficient method of passive emitter location,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 4, pp. 1091–1104, Oct. 1992.
- [9] J. Kaufmann and W. Hutchinson, *Emitter location with LES-9/9 using differential time of arrival and differential Doppler shift* Lincoln Lab., MIT, Lexington, MA, Rep. 698 (Rev. 1), 2000.
- [10] D. Mušicki and R. Evans, “Measurement Gaussian sum mixture target tracking,” in *9th Int. Conf. Inf. Fusion*, Florence, Italy, Jul. 2006.
- [11] D. Mušicki and W. Koch, “Geolocation using TDOA and FDOA measurements,” in *11th Int. Conf. Inf. Fusion*, Cologne, Germany, Jun./Jul. 2008.
- [12] D. Mušicki, B. La Scala, and R. Evans, “The integrated track splitting filter—Efficient multi-scan single target tracking in clutter,” *IEEE Trans. Aerospace Electron. Syst.*, vol. 43, no. 4, pp. 1409–1425, Oct. 2007.
- [13] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Norwood, MA: Artech House, 1999.
- [14] J. L. Williams and P. S. Mayback, “Cost-function-based Gaussian mixture reduction for target tracking,” in *5th Int. Conf. Inf. Fusion*, Fusion 2003, Cairns, Queensland, Australia, Jul. 2003, pp. 1047–1054.
- [15] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter*. Norwood, MA: Artech House, 2004.

## **AUTHORS’ BIOGRAPHY**



**Aous Y. Ali**, received the B.Sc. degree in Communication Engineering from Iraq, Mosul, in al Mosul University in 2009, and the M.Sc. degree in Electronic and Communication Engineering in SHIATS University in 2012. His research interests include Data communication and Telecommunication. Wireless communication systems, Mobile communication, control engineering, signal processing and Fuzzy logic.



**Ibrahim Beram Jasim** received B.Sc. degree in Electronic and Communication Engineering from Iraq, MOSUL University in 2010 and M.Sc in Electronic and Communication Engineering from Sam Higgin bottom University in December 2012 his research interest include telecommunication, data communication, trucking model system, signal processing and control engineering.